PUBLIC SPENDING AND ECONOMIC GROWTH

di Renato Balducci[1]

Abstract

My intention in this brief essay is to verify whether the results obtained by Barro (1990) and by Alesina and Rodrick (1994) in relation to the influence of public investments on the economy's rate of growth are also confirmed when a share of public spending is allocated to public consumption in the economy's utility function. Introducing a positive externality on private consumption into the intertemporal optimization problem seemingly generates less unequivocal results about the role of public spending policies. The latter no longer exert an effect on the growth of the economy solely through the positive externality in production induced by public investments; they also operate through a further channel which consists of consumption decisions and is therefore influenced both by the degree of substitutability between public and private consumption, and by the impatience to consume of households.

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1. Introduction

New Growth Theory[2] has sought to recover – within the framework of the rational and optimizing behaviour of agents – the scope for the allocative intervention of the policy-maker that was undervalued or indeed denied by the previous neoclassical theory of growth.[3] Despite the differences of analytical structure among the various models of endogenous growth, and despite the distinctiveness of their founding hypotheses, the feature shared by them all is that they explain balanced growth at a positive and constant *endogenous rate*. At the basis of this important property there is always some form of non-decreasing returns to scale due to constant marginal return in one or more accumulable and reproducible factors. The basic hypothesis of Barro's model (1990)[4] is that the government, having abandoned traditional public consumption spending policies, furnishes production services in quantities equal to $z(t)$ per unit of labour and finances such expenditure with proportional taxes on income : $z(t)$ =

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[2] The authors and studies that have given rise to New Growth Theory are the following: R. M. Romer (1986), (1990); R. Lucas (1988); R. Barro (1990); S. T. Rebelo (1991); P. Aghion and P. Howit (1992).
[3] As regards neoclassical growth theory, reference is made principally to Robert Solow's (1956) model. On the economic policy implications of new growth theory see the following works: M. F. Scott (1992); K. G. Shaw (1992).
[4] R. J. Barro (1990), *op. cit.* See also Barro and Sala-I-Martin (1992), (1995). Barro assumes that the government purchases a portion of the private output and then uses these purchase to provide free public services to private producers. He considers the provision of infrastructure services, activities that maintain property rights, such as police services and national defence, and so on. Therefore, the government purchases is non rival and non excludable, as a public good.

τy(t)). Hence *(1 - τ)y(t)* is the income available to the economy for investments and consumption.

The diverse nature of public spending affects the form of the production function, which is of Cobb-Douglas type with constant returns to scale in the two accumulable production factors, *k(t)* and *z(t),* and concave with respect to each of them.

If the economy is aware of the public budget constraint: *z(t) = τy(t))* and if it perfectly knows that the government will comply with that constraint, then we can straightforwardly obtain the following production function at constant returns to scale in the accumulable factor,[5] i.e the stock of private capital:

$$y(t) = Ak(t)^{\alpha} z(t)^{1-\alpha} = Ak(t)^{\alpha} (\tau y(t))^{1-\alpha} = ak(t)\tau^{\frac{1-\alpha}{\alpha}}$$

where *a=Aexp(1/α).*

Solving the intertemporal optimum problem of an economy that maximizes the current value of the flow of future utilities, the steady-state growth rate of output per unit of labour is the following:

$$g = \frac{1}{\sigma}\left[(1-\tau)y_k - \rho\right] = \frac{1}{\sigma}\left[a(1-\tau)\tau^{\frac{1-\alpha}{\alpha}} - \rho\right] = g_{barro}$$

where (1/σ) is the intertemporal elasticity of substitution and ρ is the intertemporal discount rate.

One can easily ascertain that the rate of income growth per unit of labour, in steady-state conditions, is influenced by *public spending on production services*. An increase in the tax rate τ reduces the income available for consumption and private investment, and therefore slows the pace of the economy's growth. At the same time, however, an increase in τ translates into an increase in public services *z(t)* to firms – an increase which grows in proportion to the marginal productivity of capital. Which of the two effects will prevail depends on the functional form of the production function. In the case examined here, an increase in the tax rate boosts the growth rate of the economy until $\tau < 1\text{-}\alpha$, and it reaches the maximum when $\tau = 1\text{-}\alpha$.[6] Above this threshold, an increase in the tax rate reduces the growth rate of the economy because of the effect brought about by the reduction in disposable household income. A natural extension of this important finding has been suggested by Barro himself, who argues that all public expenditures that produce externalities generalized to the firms' system – such as the defence of property rights, spending on justice, defence, etc. – may have a positive impact on the economy's growth rate. Conversely, because spending on public consumption reduces income net of taxes, it slows the economy's equilibrium growth rate.[7]

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[5] That is, an 'AK' production function as defined by S. T. Rebelo (1991), *op. cit.*

[6] See Barro (1990), *op.cit.,* p. 109.

[7] See Barro and Sala-I-Martin (1992). Figures from 98 countries in the 1960-95 period show a negative and significative relationship between public consumption and per capita growth rate of GNP.

This line of analysis has been reprised in a study by Alberto Alesina and Dani Rodrick (1994)[8] in whose endogenous growth model the key role is performed by the conflict between workers and capitalists. Alesina and Rodrick assume that the economy comprises workers who entirely consume their incomes, each of them inelastically supplying one unit of labour, and capitalists who possess the capital stock, whence derives their income which they partly consume and partly save. The (Nash equilibrium) solution of the distributive conflict, obtained by modelling the government's fiscal policies, determines the growth rate of the economy. The government decides both the tax rate on capital τ and the allocation of public spending between lump sum transfers to workers, which are equal to $(1-\eta)$ of the tax yield, and public investments, $z(t) = \eta\tau k(t)$, which act as input into the aggregate production function, generating a linear relation between per capita output and per capita capital similar to the one obtained in Barro's (1990) model. Analogously, the equilibrium growth rate $g(\eta, \tau)$ is a constantly increasing function of the proportion η of productive public spending, and concave with respect to the tax rate.

If the government's policies are intended to maximize the growth rate of the economy, they are only of benefit to capitalists. Instead, if they are also intended to protect the well-being of workers, they are unable to secure the maximum rate of growth. Finally, if the government selects policies as if it were the median voter in a democratic system,

[8] A. Alesina and D. Rodrick (1994). See also A. Alesina and R. Perotti (1996).

it can be shown that the lower the growth rate, the more unequal the distribution of wealth. In other words, *democracies with a fairer distribution of wealth are able to achieve better rates of growth*.

The hypothesis that I wish to examine is that the results obtained by Barro (1990) and by Alesina and Rodrick (1994) are also confirmed when a share of public spending is allocated to public consumption in the economy's utility function[9]. In effect, the introduction into the intertemporal optimization problem of a positive externality on private consumption seemingly generates less unequivocal results about the role of public spending policies. The latter no longer exert an effect on the growth of the economy solely through the positive externality in production due to public investments; they also operate through a further channel which consists of consumption decisions and is consequently influenced both by the degree of substitutability between public and private consumption, and by the impatience to consume of households.

2. A model of endogenous growth with public spending

2.1. In order to verify the above hypothesis, I shall construct a simple model of endogenous growth similar to the one originally developed by Barro (1990), my purpose being to reproduce that model's results and compare them with the more general ones yielded by my model.

To this end, I assume that the government levies income tax at proportional rates, $0 \leq \tau < 1$, with a yield of $\tau y(t)$. A share $0 < \eta \leq 1$ of the yield is allocated to public investments which increase the efficiency of private productive capital[10]; the remaining share, $(1-\eta)$, is used to furnish the economy with public consumption that increases the utility of private consumption and partly substitutes it[11].

Disposable household income is spent on private consumption and investment. In per capita terms:

$$\dot{k}(t) = (1-\tau)y(t) - c(t) - nk(t) \tag{1}$$

where n denotes the investment rate necessary to endow each new individual with a capital stock equal to that available to already-existing individuals.

The production function, expressed in terms of labour units, is a Cobb-Douglas with constant returns to scale in private capital and public investment. If the economy is aware of the public budget constraint: $z(t) = \tau y(t))$, and if it certainly knows that the government will comply with that constraint, we can straightforwardly obtain the following production function at constant returns to scale in the accumulable factor, namely private capital $k(t)$:

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[9] In effect, Barro and Sala-I-Martin (1995), p.152, write : "We could also allow for public consumption services as an influence on households' utility. If these services enter separably from c in the utility function, then these activities influence growth in the model only if the expenditures are financed by a distorting tax"

[10] As in Barro(1990), I assume that the government purchases a portion of the private output and uses it to provide free public services to private producers (infrastructure services, protection of property rights, such as police services and national defence, and so on).

2 $$y(t) = Ak(t)^{\alpha}(\eta\tau y(t))^{1-\alpha} = ak(t)(\eta\tau)^{\frac{1-\alpha}{\alpha}}$$

where $a=A\exp(1/\alpha)$, $y_k=y(t)/k(t)$, $y_\tau= ((1-\alpha)/\alpha)y(t)/\tau$, $y_\eta= ((1-\alpha)/\alpha)y(t)/\eta$

The utility function is hypothesised as of CES type, its arguments being per capita consumption $c(t)$ and per capita public consumption $(1-\eta)\tau y(t)$, which may both increase the utility of private consumption and partly substitute for it:

3 $$u(t) = \left[(1-\sigma)c(t)^{\gamma} + \sigma((1-\eta)\tau y(t))^{\gamma}\right]^{\frac{1}{\gamma}} = \left[(1-\sigma) + \sigma T(\eta,\tau)^{\gamma}(\frac{k(t)}{c(t)})^{\gamma}\right]^{\frac{1}{\gamma}} c(t)$$

where $-\infty \leq \gamma \leq +1$,

$\varphi = \frac{1}{1-\gamma}$, $0 \leq \varphi \leq +\infty$, represents the substitution elasticity between the two goods;

$$T(\eta,\tau) = (1-\eta)a\tau(\eta\tau)^{\frac{1-\alpha}{\alpha}}$$

$$u_c(t) = (1-\sigma)\left[(1-\sigma) + \sigma T(\eta,\tau)^{\gamma}\left(\frac{k(t)}{c(t)}\right)^{\gamma}\right]^{\frac{1-\gamma}{\gamma}}$$

$$u_k(t) = \sigma\left[(1-\sigma) + \sigma T(\eta,\tau)^{\gamma}\left(\frac{k(t)}{c(t)}\right)^{\gamma}\right]^{\frac{1-\gamma}{\gamma}} T(\eta,\tau)^{\gamma}\left(\frac{k(t)}{c(t)}\right)^{\gamma-1}$$

[11] I consider, as in Barro and Sala-I-Martin (1995) that enters into the household utility function the public consumption services excluding expenditures for the national defence and for public education.

The intertemporal maximization problem requires us to maximize the current value of future flows of utility, opportunely choosing the optimal consumption path *c(t)* (control variable) in respect of (i) the dynamic constraint constituted by the macroeconomic equilibrium condition (1); (ii) the non-negativity conditions of the capital stock and consumption: $k(t) \geq 0$, $c(t) \geq 0$; and (iii) the positivity condition of the initial capital stock: $k(0)=k_0 > 0$.

We also assume that the economy considers as given and constant both the tax rate τ and the distribution of public spending between productive investments η and public consumption $(1-\eta)$.

The Hamiltonian function of this problem assumes the following form:

$$H[k(t), c(t), \mu(t), \tau, \eta] = e^{-\theta t}u(t) + \mu(t)[(1-\tau)y(t) - c(t) - nk(t)]$$

where $\theta = \rho - n > 0$ represents the intertemporal discount factor,[12] and the *costate* variable $\mu(t)$ expresses the present value, measured in terms of marginal utility of consumption, of one additional unit of private capital at time t.

[12] In solving the optimum problem, the units of population at the initial date are normalized to 1: N(0)=1.

Now I introduce the first-order maximum conditions,[13] which are obtained by deriving the Hamiltonian function with respect to the control variable *c(t)* and the state variable *k(t)*:

4 $$\frac{\partial H(.)}{\partial c(t)} = e^{-\theta t} u_c(t) - \mu(t) = 0$$

5 $$-\frac{\partial H(.)}{\partial k(t)} = \dot{\mu}(t) = -e^{-\vartheta t} u_k(t) - \mu(t)\left[(1-\tau)(y(t)/k(t)) - n\right]$$

It is also necessary to impose the limit condition: $\lim_{t \to \infty} \mu(t)k(t) = 0$ which states that whatever is left at the end of the (infinite) time horizon must be of zero value because utility cannot accrue from the bequest.

We may now rewrite (4) as follows:

4a $$e^{-\vartheta t}\left[(1-\sigma) + \sigma T(\eta\tau)^{\gamma}\left(\frac{k(t)}{c(t)}\right)^{\gamma}\right]^{\frac{1-\gamma}{\gamma}} = \frac{\mu(t)}{1-\sigma}$$

and substitute in (5) to obtain the rate of variation of the costate variable:

[13] For techniques to solve intertemporal optimum control problems see R. Balducci, G. Candela, A.E. Scorcu, *Teoria della politica economica. Modelli dinamici e stocastici,* Zanichelli , Bologna 2003, chap.13 and chaps. 17-18.

5a $$-\frac{\dot{\mu}(t)}{\mu(t)} = -g_{\mu}(t) = \frac{\sigma}{1-\sigma} T(\eta,\tau)^{\gamma}\left(\frac{c(t)}{k(t)}\right)^{1-\gamma} + B(\eta,\tau)$$

where $B(\eta,\tau) = a(1-\tau)(\eta\tau)^{\frac{1-\alpha}{\alpha}} - n > 0$[14]

The interpretation of these conditions is as follows. Equation (4) defines the optimal time path of the control variable *c(t)*, setting it in relation to the time path of the costate variable $\mu(t)$. Recall that the latter variable, as a shadow price, indicates the value attributed to the increase in the capital stock (the state variable), i.e. to the investment that competes with consumption in the use of produced income. Consequently, equation (4) indicates that the present value of the marginal utility of consumption, in steady-state equilibrium, must be equal to the present value of one additional unit of value expressed by the costate value $\mu(t)$.

Equation (5) states that the optimal rate of saving, and therefore of investment, equivalent to the opposite of the rate of variation in the costate variable $-g_{\mu}(t)$, in steady-state equilibrium, must be equal to the marginal productivity of capital minus the population growth rate in order to maintain capital per worker *k(t)* constant along the balanced growth path.

[14] The value of B(.) must be positive in order to allow for at least non-negative private consumption. In fact, B(.) represents the average (and marginal) per capita income (net of the investment necessary to endow new workers with the same stock of capital as possessed by already-employed workers) which serves to satisfy both per capita consumption needs and to increase the per capita endowment of capital (net investment).

From the economy's budget constraint, equation (1), we can obtain the definition of the growth rate of the per capita capital stock:

6 $$\frac{\dot{k}(t)}{k(t)} = g_k(t) = -\frac{c(t)}{k(t)} + B(\eta,\tau)$$

Because in steady state the rate of growth of the capital stock must be constant, and because all the policy variables (η, τ) are deemed constant, by differentiating (6) with respect to time, one can demonstrate that in balanced growth conditions it must be that:

7 $$\frac{\dot{c}(t)}{c(t)} = \frac{\dot{k}(t)}{k(t)} \Rightarrow g_c = g_k = \text{costant}$$

I again apply condition (4a), transcribing it in logarithmic form:

4b $$-\theta t + \ln(1-\sigma) + \frac{1-\gamma}{\gamma}\ln\left[1-\sigma+\sigma T(\eta,\tau)^{\gamma}\left(\frac{c(t)}{k(t)}\right)^{-\gamma}\right] = \ln\mu(t)$$

Therefore, deriving it with respect to time and adjusting the terms, we obtain the following relation between rates of variation:

8 $$-\theta-(1-\gamma)\frac{\sigma T^{\gamma}\left(\frac{c(t)}{k(t)}\right)^{-\gamma}}{1-\sigma+\sigma T^{\gamma}\left(\frac{c(t)}{k(t)}\right)^{-\gamma}}\left(g_c(t)-g_k(t)\right)=g_\mu(t)$$

From this relation, taking account of (7), we obtains the steady-state rate of growth of the costate variable:

9 $g_{\mu}(t) = -\theta$

Substituting (9) in (5a) yields the optimal value of the ratio between consumption and capital:

10 $$\frac{c(t)}{k(t)} = \left[(\theta - B(\eta,\tau)) \frac{1-\sigma}{\sigma} T(\eta,\tau)^{-\gamma} \right]^{\frac{1}{1-\gamma}}$$

Finally, substituting (10) in (6) yields the rate of growth of the per-capita capital stock, which in steady-state conditions is constant and equal to the rates of growth of consumption g_c and per capita product g_y:

11 $$g = B(\eta,\tau) - \left[(\theta - B(\eta,\tau)) \frac{1-\sigma}{\sigma} T(\eta,\tau)^{-\gamma} \right]^{\frac{1}{1-\gamma}} = g_c = g_k$$

where $$sign \frac{\partial g}{\partial \gamma} = -sign \left[\gamma(1-\sigma) \left((1-\tau) a\tau(\eta\tau)^{\frac{1-\alpha}{\alpha}} - \rho \right) + \sigma(1-\eta) a\tau(\eta\tau)^{\frac{1-\alpha}{\alpha}} \right]$$

Recall that the range of significant variation of γ is between -∞ and +1. Consequently, the substitution elasticity between private and public consumption varies between perfect complementarity, γ=-∞ ➔ φ= 0, and perfect substitutability, γ=1 ➔ φ= +∞. Finally, for γ=0 ➔φ= 1, the CES collapses into the Cobb-Douglas utility function.

It is thus evident that, for $\gamma \geq 0$, the expression in the square brackets is positive, and consequently that the growth rate diminishes with an increase in the degree of substitutability between private consumption and public consumption. When γ is instead negative, and large in absolute value, the sign of the derivative is reversed: that is, the growth rate increases with an increase in γ.

Let us first consider the case $\gamma=1$, where the substitution elasticity between the two consumption goods becomes infinite, so that it is not possible to define an optimum ratio between public and private consumption, i.e. an internal solution of the problem. One instead has a corner solution. As a consequence, it is also not possible to establish the optimum growth rate except as an asymptotic solution.

Under quite plausible conditions, whereby $\frac{(1-\sigma)(B(\eta,\tau)-\theta)}{\sigma T(\eta,\tau)} < 1$, the limit of (11) for $\gamma \to 1$ is the following:

(11a) $$g(\gamma \to 1) = B(\eta,\tau) = a(1-\tau)(\eta\tau)^{\frac{1-\alpha}{\alpha}} - n$$

which is an increasing function of η and a concave one of τ, reaching its maximum for $\tau=(1-\alpha)$. In this case, the corner solution foresees nil private consumption.

By contrast, when $\gamma=-\infty$, the CES represents a utility function with perfectly complementary goods, i.e. with nil substitution elasticity. In this case, the growth rate obtained contains no reference to the impatience to consume of households, given that the types of consumption are combined in a fixed ratio. Only the policy variables that affect production matter in defining the growth rate:

12 $$g(\gamma = -\infty) = a\tau(\eta\tau)^{\frac{1-\alpha}{\alpha}}(\eta - \tau - n)$$

It is evident in this case that the growth rate is positive only if η-n >τ, i.e. if the share of the public budget allocated to public investments is sufficiently large, or in any case greater than the proportional tax rate. Moreover:

$$sign\frac{\partial g(\gamma = -\infty)}{\partial \tau} = sign[\eta - n - \tau(1+\alpha)]$$

$$sign\frac{\partial g(\gamma = -\infty)}{\partial \eta} = sign[\eta - (n+\tau)(1-\alpha)]$$

That is, the growth rate is an increasing function of public investments and a concave function of the tax rate τ; g increases until τ<(η-n)/(1+α) and diminishes after exceeding that value until it becomes negative for τ<(η-n).

Let us finally consider the case where γ=0. By reproducing a Cobb-Douglas utility function, this enables more immediate comparison with the results obtained by Barro (1990), in particular as regards the explicit formulation of the growth rate:

13 $$g(\gamma=0)=\frac{1}{\sigma}\left[(1-\tau)a\tau(\eta\tau)^{\frac{1-\alpha}{\alpha}}-\left(\sigma n+(1-\sigma)\rho\right)\right]$$

It follows straightforwardly from (13) that the economy's growth rate is a concave function of the tax rate which is increasing for $\tau < 1\text{-}\alpha$ and reaches its maximum for $\tau = 1\text{-}\alpha$, as amply demonstrated by Barro (1990). Moreover, g is an increasing monotonic function with respect to the share η of the tax yield allocated to public investments, so that the highest growth rate is obtained at the upper extreme of the range of definition of that variable, i.e. for $\eta = 1$.

Hence, for $\tau = \tau° = 1\text{-}\alpha$, $\eta = \eta° = 1$, the maximum growth rate of the economy[15] is accomplished by more appropriate economic policies targeted on development (under the hypothesis of the government as leader)

14 $$g(\eta°,\tau°)=\frac{1}{\sigma}\left[a\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}-(\sigma n+(1-\sigma)\rho)\right]$$[16]

[15] Should we want to rationalize this result in terms of games theory, we may say that the government acts as leader and the economy as follower, in Stackelberg's sense. In fact, the economy maximises its functional objective, taking as given the strategies (concerning τ and η) implemented by the government. The latter instead knows the economy's reaction function (i.e. g(τ, η)), subsequently maximising its value.

[16] Note that if we assume the population to be constant, when comparing (10) with the equation that defines Barro's (1990) growth rate, the only difference is the term in the square brackets: +σρ, which is entirely due to the effect of public consumption on the utility function.

The difference with respect to the analogous maximum growth rate obtained by Barro (1990) is the following:

$$g(\eta^{\circ},\tau^{\circ}) - g_{Barro} = \theta = \rho - n > 0$$

That is to say, consideration of a further effect of public spending on the economy's utility function permits a more sustained pace of growth, the tax rate remaining equal. This contradicts Barro's assertion that spending for public consumption, by reducing disposable household income, slows the economy down. I have shown, indeed, that *spending for public consumption may also be a valid means to promote growth*, one which becomes the more efficacious the greater the intertemporal discount rate – that is, the greater the 'haste' of households to consume. The reason for this is as follows: the availability of public consumption at least partly substitutes private consumption, enabling households to save and to invest more in productive capital. This effect will be the more robust, the greater the haste of households to consume.

2.2. The foregoing solution of the problem of the intertemporal optimization of the economy has been performed under the following hypotheses: (i) the economy perfectly knows and takes account of the public-sector budget constraint; (ii) the tax rate τ and the distribution of public spending between public investment η and public consumption $(1-\eta)$ are considered to be given and constant.

Let us now drop hypothesis (ii), imagining that the electoral choices of the economy – at least in a long-period framework coherent with the treatment of growth problems – may induce governments to establish optimal values for the tax rate τ^* and for the distribution η^* of public spending. That is to say, the government may fix by law exactly those values which maximise the present value of the future flows of utility deriving from both private consumption and public consumption.[17]

For this purpose we must zero-set the prime derivatives of the Hamiltonian function with respect to the policy variables τ and η, taking account of equation (4), which defines the optimal path of private consumption *c(t)*. With the appropriate simplifications,[18] we may write these two first-order conditions thus:

15 $H(...)_{\tau} = 0 \Rightarrow \sigma c(t) + (1-\sigma)y(t)(1-\alpha-\tau)=0$

16 $H(...)_{\eta} = 0 \Rightarrow \sigma c(t)(1-\alpha-\eta) + (1-\sigma)y(t)(1-\eta)(1-\alpha)=0$[19]

[17] If we wish to state the problem in game theory terms, we may say that the government and the economy have the same utility function and *cooperate* to achieve the common maximum of the same functional objective. This is therefore a *cooperative game* undertaken by a hypothetical benevolent planner.

[18] This case is studied under the hypothesis of a Cobb-Douglas utility function, which is a particular instance of the CES function for γ=0, because under this hypothesis it is possible to make immediate comparison with Barro's (1990) results.

[19] These conditions are transcribed on the hypothesis that the costate variable μ(t) does not have zero value for all t, in which case the problem would lose its nature as an intertemporal optimization problem and would pertain to a static context.

Resolving the two conditions (15) and (16) with respect to the ratio c(t)/y(t), and equalizing them, we obtain the following relation between the tax rate and allocation of the tax yield:

17 $\eta = (1-\alpha)(\alpha+\tau)/\tau$

Finally, combining (15) with the value of the consumption/capital ratio (10), taking account of (17), we obtain the following values for the policy variables:

18 $\tau^* = Z(\rho) - \alpha$

19 $\eta^* = (1-\alpha)\, Z(\rho)/(Z(\rho) - \alpha)$,

19a $\tau^*\eta^* = (1-\alpha)\, Z(\rho)$

where $Z(\rho) = \left[\frac{\rho}{a\alpha(1-\alpha)}\right]^{\frac{\alpha}{1-\alpha}}$, $Z_\rho > 0$

It is clear that if $Z(\rho)=1$, i.e. if it were the case that $\rho = \alpha\alpha(1 - \alpha)$, the optimum values of the tax rate τ^* and of the share allocated to public investments η^* would exactly correspond to the ones which maximise the growth rate as defined by equation (10).

In general, however, $Z(\rho)\neq 1$. Consequently, the optimum or desired growth rate $g(\tau^*,\eta^*)$ differs from the maximum rate $g(\tau°,\eta°)$:

20 $$g(\tau^*,\eta^*)=\frac{1}{\sigma}\left[a(1+\alpha-Z(\rho))(1-\alpha)^{\frac{1-\alpha}{\alpha}}T(\rho)^{\frac{1-\alpha}{\alpha}}-(\sigma n+(1-\sigma)\rho)\right]=g^*$$

Per capita output, obtaining by choosing the optimal values of the policy variables, increases at a constant rate g* which depends on the *foundamentals* of the economy, and in particular on the average productivity *a* of composite capital; on the intertemporal discount rate ρ; and on the inverse of the substitution elasticity between present and future consumption σ.

It is interesting to verify under what conditions the optimum or desired growth rate $g(\tau^*, \eta^*)$ differs from the maximum rate $g(\tau°, \eta°)$:

$$\Delta(\rho)=g(\tau°,\eta°)-g(\tau^*,\eta^*)=\frac{a}{\sigma}(1-\alpha)^{\frac{1-\alpha}{\alpha}}\left[\alpha-(1+\alpha-Z(\rho))Z(\rho)^{\frac{1-\alpha}{\alpha}}\right]$$

where

$$\Delta_\rho=\frac{1}{1-\alpha}\left[Z(\rho)+\alpha^2-1\right]\geq\leq 0 \quad \text{se} \quad \frac{\rho}{a}\geq\leq\alpha(1-\alpha)(1-\alpha^2)^{\frac{1-\alpha}{\alpha}}$$

Firstly, it is evident that if $Z(\rho)=1$, then $\Delta(\rho)$ would be zero; that is, the optimum growth rate would correspond to the maximum rate, as one would expect on logical grounds. For $Z(\rho)\neq 1$, the gap between the two growth rates is explained by the ratio between the value of the intertemporal discount rate ρ, i.e. the haste to consume, and the value of the productivity *a* of composite capital (private and public). Because $\Delta(\rho)$ is non-linear, there may be intervals in the parameter values which reverse the inequality relation between the two growth rates[20].

As will be seen from Figures 1 and 2, constructed on data from Table 1, there is a value of ρ so low that the optimum tax rate is nil.[21] In other words, when households are very 'patient', so that they evaluate more immediate consumption and more distant consumption in substantially the same way, it is optimal for the economy that the government is 'absent'; and in the absence of the government the two growth rates $g(\eta°,\tau°)$ *e* $g(\tau^*,\eta^*)$ coincide.

For higher values of ρ, as households' impatience to consume increases, the economy is willing to pay a positive tax rate lower than $\tau°$: $\tau^* < \tau°=1-\alpha$, and accepts that the entire tax yield may be allocated to supporting productive investments, $\eta^*=1$. The resulting growth rate is less than the maximum potential one: $\Delta(\rho)<0$.

As impatience to consume increases further, when the intertemporal discount rate reaches the value $\rho^*= a\alpha\,(1-\alpha)$, then $\Delta(\rho)=0$; that is, the two growth rates once again coincide because the desired tax rate is exactly the same as the one which maximises the

[20] According to the value of α, the equation which defines the difference between the two growth rates may be more than grade 2 (as hypothesised in Figure 2), so that there may be more than two zeroes.

[21] For $\rho_{\min} = a(1-\alpha)\alpha^{\frac{1}{\alpha}}$, one obtains $\tau^* = 0$ and η^* is indeterminate; for $\rho_{\max} = a\alpha(1-\alpha)(1+\alpha)^{\frac{1-\alpha}{\alpha}}$, one obtains that $\tau^*=1$, $\eta^* = 1-\alpha^2$

growth rate: $\tau^* = \tau° = 1 - \alpha$, and public spending is entirely devoted to public investments: $\eta^* = \eta° = 1$.

When this value of the intertemporal discount rate has been exceeded, i.e. in the presence of households extremely impatient to consume, the economy is willing to accept a tax rate τ^* greater than $\tau° = 1 - \alpha$, and a growth rate $g(\tau^*, \eta^*)$ below the maximum rate $g(\tau°, \eta°)$, provided that a share of the tax yield is allocated to public consumption, $\eta^* < 1$.

Table 1
Parameter values: ***a=0,2*** **,** $\boldsymbol{\alpha = 0.5}$**,** $\boldsymbol{\sigma = 0.8}$**,** ***n=0,01***

(*Percentage values*)

ρ	τ*(ρ)	η*(ρ)	g(η°,τ°),	g(τ*,η*)	Δ(ρ)
2.5	0.00	Indeterminato	4.375	4.375	0.000
3.0	10.0	1.00	4.25	4.75	-0.50
4.0	30.0	1.00	4.00	4.75	-0.75
5.0	50.0	1.00	3.75	3.75	0.000
6.0	70.0	86.0	3.50	1.75	1.75
7.0	90.0	78.0	3.25	-1.25	4.50
7.5	100.0	75.0	3.15	-3.125	6.275

Figure 1 The relations between the discount rate ρ and the taxation rate τ (continuous line) and the share of public investment η (dot line).

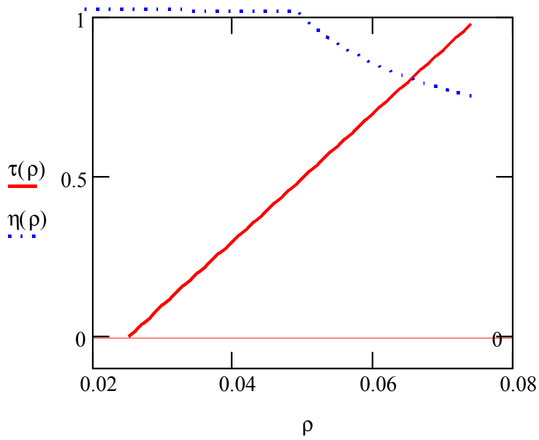
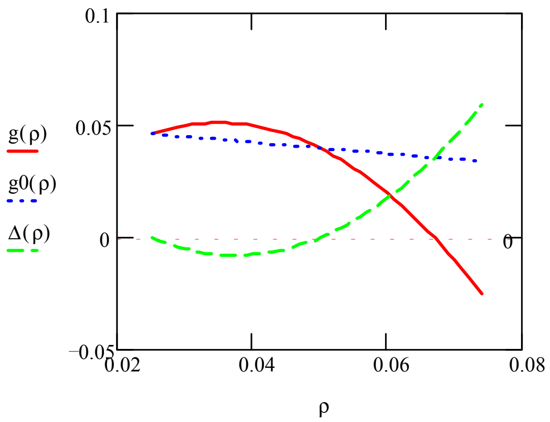


Figure 2 The relations between the discount rate ρ and the optimal growth rate g(ρ) (continuous line) , the maximum growth rate g0(ρ) (dot line) and the difference between them Δ(ρ) (dash line).



3. Conclusions

The foregoing exercise has enabled me to verify the robustness of Barro's (1990) result that the most powerful instrument with which governments can influence the growth rate of the economy is public spending on productive investments. The latter interact with private productive capital to increase its productivity, giving rise to a positive externality. Composite capital consisting of private capital and public investments displays constant returns to scale. Consequently, the economy is able to grow, in steady state, at a constant rate positively correlated with the proportional tax rate τ until the latter is less than, or at most equal to, the contribution $(1-\alpha)$ made by public investments to per capita product. In other words, from a neoclassical perspective tax represents the fair price 'paid' for the production factor of public investment. Above this threshold, an increase in the tax rate reduces the growth rate of the economy because of the prevalence of the effect due to the decline in disposable household income (taxation effect).

This result is rather robust, for it is also confirmed, albeit as a particular case, when Barro's (1990) model is generalized by hypothesising that a share of public spending is allocated to public consumption, which interacts with private consumption to increase its utility. This twofold characterization of public spending opens a further channel for influence on the growth rate which operates through the generation of a positive externality on consumption. This possibility is *per se* sufficient to produce a potential, or maximum, growth rate $g(\eta°, \tau°)$ which is always greater than the analogous growth rate obtainable with Barro's model, g_{Barro}. The difference between the two rates is due to the intertemporal discount rate (net of the population growth): ρ-n; and it is greater, the more impatient the economy is to consume. Hence, applying the same tax rate and allocating public spending in the same way, it is possible to obtain more sustained growth than envisaged by Barro's model (1990).

However, once this further possibility for allocation of public spending has been introduced, one may enquire as to the optimal values of both the tax rate and the allocation of the yield between productive investments and public consumption. The question can obviously only be asked if we imagine that, in the long run, the government adopts the economic policies most desired by the economy; or in other words, if the government and the economy cooperate with each other in pursuit of the same objective.

If we define the optimal values, from the economy's point of view, of η^* and τ^*, it is evident that the tax rate deriving from them $g(\tau^*,\eta^*)$ may diverge from the potential one $g(\tau^\circ,\eta^\circ)$. Interestingly, the gap between the maximum or potential growth rate and the desired rate is not always positive or always negative; rather, it exhibits alternate signs in relation to the ratio between the economy's impatience to consume ρ and the productivity of composite capital a. The potential growth rate is greater than the desired one when the economy is both very patient and very impatient to consume. In the latter case, the economy is willing to accept a high level of taxation as long as a share of the yield is allocated to public consumption, thereby slowing down growth. There are therefore (at least) two situations in which the desired and the potential growth rates coincide. The first is the rather banal case in which the tax rate is nil, i.e. when the government does not interfere in economic affairs. The second case is the more interesting one in which it is the tax rate that maximises the growth rate as identified by Barro (1990): $\tau^{\circ}=1-\alpha$.

Finally, for intermediate values of the intertemporal discount rate, the desired growth rate is greater than the potential rate even with tax rates below τ°. If this is in fact the case, governments could foster the growth of the economy without being *excessively* invasive in the market economy.

References

Aghion P. e Howit P.
1992, *A Model of Growth trough Creative Desctruction,* in "Econometrica", 60, pp.323-351

Balducci R., Candela G., e Scorcu A.E.
2002, *Teoria della politica economica. Modelli dinamici e stocastici*, Zanichelli, Bologna,

Barro R.J.
1990, *Goverment Spending in a Simple Model of Endogenous Growth,* in "Journal of Political Economy", 98, pp.103-125

Barro R.J. , Sala-I-Martin X.
1992, *Public Finance in Models of Economic Growth*, in "Review of Economic Studies", 59, 4, pp.645-661

Barro R.J. , Sala-I-Martin X.
1995, Economic Growth, McGraw-Hill, N.Y.

Lucas R.
1988, *On the Mechanics of Economic Development,*in "Journal of Monetary Economics", 22, pp.3-42

Rebelo S.T.
1991, *Long-Run Policy Analysis and Long-Run Growth",* in "Journal of Political Economy", 99,pp.500-521.

Romer P.M.
1986, *Increasing Returns and Long-Run Growth,* in "Journal of Political Economy", 94, pp.1002-1037

Romer P.M.
1990, *Endogenous Technological Change,* in "Journal of Political Economy", vol.98, pp.71-101

Romer P.M.
1994, *The Origins of Endogenous Growth,* in "Journal of Economy Perspective", vol. 8, pp.3-22

Scott M.F.
1992, *Policy Implications of 'a New View of Economic Growth',* in "Economic Journal",102, pp.622-632;

Shaw K.G.
1992, *Policy Implications of Endogenous Growth Theory,* in "Economic Journal",102, pp.611-621.

Solow R.
1956, *A Contribution to the Theory of Economic Growth,* in "Quarterly Journal of Economics, 70, pp.65-94