Short-run bargaining, factors shares and growth [1]

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Abstract

We assume that, in the short run, firms and unions bargain efficiently on wages and employment and determine the labour and capital shares. Using an endogenous growth model based on human capital accumulation, and on the hypothesis that firms invest profits in physical capital while workers optimally allocate their earnings between consumption and investment in human capital, we determine the wage rate and the labour share that maximizes individual expected utility. Our main result is that the optimal labour share must be higher than the one arising from perfect competition in the labour market. Therefore, trade unions are necessary for optimal economic growth.

1 Introduction

During the 1950s and 60s, the economic literature studied the complex relationship between income distribution and economic growth. This issue was debated, amongst others, by Kaldor (1956), Pasinetti (1962, 1969) and Samuelson - Modigliani (1966). Attention focused mainly on the different propensities to save of the social classes comprising workers and capitalists, and on the change in the average value of the rate of saving brought about by variation in the proportions of total income accruing to one or other class.

A branch of growth theory focuses on the distributive conflict that takes place in non-competitive labour markets. It has long been recognized that trade unions are able to influence capital accumulation through their involvement in the fixing of wage and employment levels. In a competitive firm, higher wages lead to the substitution of labour by capital and to a fall-off in production. The overall effect on the capital stock is therefore ambiguous.

In a situation in which companies and unions bargain over both wages and employment, in the absence of binding agreements between the parties it is likely that the incentive to invest will diminish (Grout (1984), Van del Ploeg (1987)): in fact, a larger capital stock and greater labour productivity will induce the unions to demand higher wages, thereby eroding the expected return on capital. When firms see the shortfall in their expected return, they will have less incentive to invest.

Furthermore, other theories on endogenous growth strongly emphasize that labour market institutions may influence growth. Indeed, the conclusions of some studies may differ from others simply because an assumption is changed. Gilles (2000) and Irwen and Wigger (2001) assume that the marginal propensity to save on wages is higher than that on capital, while De Groot (2001) assumes the opposite. Their conflict of views derives directly from the assumption of no dynasty versus an infinitely long dynasty. This, ultimately, determines whether the rent bearing older generation will save a lot or will not save at all.

Lingen (2002) assumes that the ratio of labour employed in the productive sector to the labour employed in the R&D sector is sticky, and that therefore higher wages in the productive sector causes unemployment in both sectors. Lingens' assumption eventually induces him to conclude that unions are growth reducing while Palokangas (2003), in contrast, concludes that real wage increases in the productive sector will increase labour supply to the R&D sector, causing growth.

Lastly, Parreño and Sánchez-Losado (1999) show that consideration of how unemployed resources are used is essential in determining effects of labour unions on growth. They show that if unemployed resources are used to educate, the unemployment caused by labour unions may be growth enhancing.

However, the results are diverse and the paths to results are also diverse, suggesting that there is no consensus on the effects of labour unions on endogenous growth. A conclusion to the argument will come when consensus is reached at each stage of building a model.

Empirical estimates based on neoclassical growth theory gave rise to an unexplained component of the rate of growth, usually called the "Solow residual". The recent economic literature has highlighted the fundamental role of human capital in explaining economic growth. The "Solow residual" is now commonly considered as the human capital contribution to growth. There is little dispute that human capital accumulation plays an important role in the growth process because education, by producing a particular form of capital based on intellectual skills, increases output.

The traditional two-factor production function of earlier growth models has gradually been substituted by a three-factor production function based on two accumulable factors (human and physical capital) and a non accumulable one (labour). Lucas's paper (1988) highlighted that the inclusion of human capital in growth models gives rise to an unresolved question on the functional distribution of income: which factor earns the part of product due to human capital? By raising the efficiency of both workers and physical capital, human capital is a sort of "public good" (justifying the public financing of educational systems) whose earnings may be appropriated by both workers and capitalists.

In order to clarify the first point, suppose the following aggregate production function, which display constant return to scale in the accumulable factors:

$$Y = b(LH)^{\alpha}K^{1-\alpha} \tag{1}$$

where Y is production, b is an exogenous scale parameter, L employment, H human capital and K physical capital.

With perfect competition in the product and the labour markets, it is usually assumed that the labour share equals to α. Thus, human capital revenue completely accrues to workers. In fact, considering human capital as a skill which results from devoting time to its acquisition (like Lucas, 1988), it is impossible to distinguish “education” from the “educated” individual worker; in this case it seems plausible to accrue to workers the total increase in productivity due to education.

> Lucas supposes there to be a set of perfectly competitive risk neutral firms producing the single good for consumers. They maximize profits and pay a wage equal to labour’s marginal product. Workers, by acquiring skills, forego some of their wage in favor of the higher wage their skill gain will command (F. Hahn, p. 9-10).

This of course seems very simplistic. In fact, higher skills and better education improve the productivity of machinery. Furthermore,

> ... intellectually skilled workers facilitate the transfer of technology...This suggests that a high level (rather than a high growth rate) of intellectual skills is associated with increase in output. If this alternative interpretation is correct, the conflict between the predicted and actual profit share may not be so easily resolved (K. Foley and R. Michl, 1999, page 173.)

Foley and Michl hypothesis seems to consider human capital as a sort of externality (and presents many characteristic of a public good) raising productivity of both labour and capital whose earnings may be not completely appropriated by workers. The trade unions operate in order to increase the revenue accruing to workers generated by this externality.

In our approach we consider both the case of workers “specific” human capital and the case of externality (even if we will concentrate on the former, postponing the analysis of the latter in appendix A). In the latter hypothesis, we will assume that the State operates through taxation in order to force individual to internalise this externality.

The aim of this article is to re-examine bargaining, functional distribution of income and endogenous growth based on human capital, following:

- the Lucas “indeterminacy” of factor shares when human capital enters the production function;
- the Kaldorian hypothesis of different propensities to save for the different social classes (or, more appropriately, earners of different types of revenue);

- the Mc-Donald, Solow (1981) idea of efficient bargaining between the social partners.

The following sections will present a two stage model on the basis of the following hypotheses:

- in the short run, the social partners (trade unions and firms) bargain efficiently at a decentralized level over wages and employment; the contract curve set the employment level as a function of the wages. Therefore, employment, production and the factor shares are determined as a function of the bargained wage. According to the Nash bargaining model, determination of the equilibrium levels of employment and wages requires the definition of a given level of bargaining power for firms and trade unions. Instead of solving the model and leaving the equilibrium dependent on an exogenous bargaining power, we prefer to leave the wage rate undetermined in the short run. Therefore the short run equilibrium is "open" in the sense of the classical and the marxian theories of *conventional* wage models.

- in the long run, households maximize their expected utility. On the hypothesis that the capital share is completely reinvested, as in the Ricardian and classical tradition[1], whereas the labour share is optimally allocated between consumption and investment in human capital[2], the optimal wage rate is determined. Therefore, in a long run perspective, households, as the owners of physical capital, decide to leave profits to firms in order to increase the physical capital stock and, as consumers and owners of human capital, decide how much to invest in human capital in order to maximize the expected utility.

We obtain the result that, in steady state, the optimal wage rate must be such that the last unit invested in physical capital and in human capital generates the same increase in the current value of the utility deriving from

[1]Modern macroeconomic theory justifies the role played by profits in explaining the investment rate through capital market imperfection which invalidate the Modigliani-Miller theorem. In particular, real profits (internal funds), allow firms to combat liquidity constraints when access to capital markets is not perfect; Chirinko (1987) , Stiglitx and Weiss (1981) Greenwald and Stiglitz, 1987 , Fazzari et al, (1988)

[2]Investment in human capital usually is measured by school enrolment, financed partly by the general taxation system and partly by households directly. Hence, the cost of schooling is mainly transferred to households.

consumption. Furthermore, there exist an optimal labour share, depending on preferences and technological parameters alone, which must be higher than the one arising from perfect competition on the labour market (the α parameter of the production function): hence, trade unions are needed to attain optimality in the economic system.

In section 2 we defines the behaviour of workers, trade unions and firms in the short run. Section 3 presents an endogenous growth model where firms invest profits in physical capital while workers optimally allocate their earnings between consumption and investment in human capital. In Section 4 we analyse the relationship between short run behaviour and long run optimality. Finally, we propose some concluding remarks.

2 The short-run bargaining

We assume decentralized efficient bargaining, in the sense that each firm and each trade union bargains jointly on wage and employment (Mc Donald, Solow, 1984).

The trade union at firm j maximizes the following union utility function:

$$W_j = L_j(w_j - v)^z$$

where v is some reference wage which we assume to be invariant across firms. As we will see later, the reference wage is a crucial determinant of the long run equilibrium; it is usually assumed to represent the last bargained wage, or the wage of some foreign "reference" country, or unemployment benefits augmented by the utility deriving by the higher leisure, and so on. We assume that the "reference" wage is higher than workers' reservation wage. The parameter z indicates the relative weight of the wage in trade unions utility. Risk neutral firms maximizes profits:

$$\Pi_j = AL_j^\alpha - w_j L_j$$

where, in the short run production function $Y(L) = AL^\alpha$, we define $A = bH^\alpha K^{1-\alpha}$.

The contract curve is given by the set of tangency points between trade unions' iso-utility and firms' iso-profit curves[3] in the space w_j, L_j, so that $\frac{W_{L_j}}{W_{w_j}} = \frac{\Pi_{L_j}}{\Pi_{w_j}}$. Equating the two slopes we obtain the contract curve at firm

[3]From the definition of union utility function and firm profit function, we obtain the

level, which can be solved for the employment level and be aggregated across firms (with a mass equal to 1) in order to obtain the aggregate contract curve:

$$L(w) = \left(\frac{\alpha z A}{(z-1)w + v} \right)^{\frac{1}{1-\alpha}} \tag{2}$$

Along the contract curve, employment is decreasing in the wage rate if $z > 1$, which represents the case where trade unions care more about wages than employment. We make a further assumption: we suppose that $z = 1$, so that trade unions maximize the wage bill. This simplification allow us to obtain a vertical contract curve, so that the employment level is now independent on the bargained wage:

$$L^* = \left(\alpha \frac{A}{v} \right)^{\frac{1}{1-\alpha}} \tag{3}$$

lower than the labour force because of our assumption of $v < w_R$, with w_R workers reservation wage. Therefore, involuntary unemployment exists.

The labour share may be written as follows:

$$q = \frac{wL^*}{Y(L^*)} = \frac{wL^*}{AL^{*\alpha}} = w\frac{L^{*(1-\alpha)}}{A}$$

Using equation 3, we easily obtain:

$$q(w) = \alpha \frac{w}{v} \tag{4}$$

Hence, $q(w)$ is increasing in the wage rate.

3 Endogenous growth and wage determination

Let us consider production, human capital and physical capital in per capita units, defining:

$$y(t) = \frac{Y(t)}{L^*} \qquad h(t) = \frac{H(t)}{L^*} \qquad k(t) = \frac{K(t)}{L^*}$$

slope of the iso-utility curve, that is: $\frac{dw_j}{dL_j} = -\frac{w_j - v}{zL_j}$ and the slope of the iso-profits curve: $\frac{dw_j}{dL_j} = -\frac{Y'_{L_j} - w_j}{-L_j}$

so that:

$$y(h,k) = Bh(t)^{\alpha}k(t)^{1-\alpha} \tag{5}$$

where $B = bL^{*\alpha} = b\left(\alpha\frac{A}{v}\right)^{\frac{\alpha}{1-\alpha}}$ is constant over time if we assume that $\frac{\dot{A}}{A} = \frac{\dot{v}}{v}$, where $\frac{\dot{A}}{A} = \alpha\frac{\dot{h}}{h} + (1-\alpha)\frac{\dot{k}}{k} \equiv g$. In the steady state equilibrium, we assume that the "reference" wage will grow at a rate g.

In the long run, the utility of each household depends on consumption level, $C(t)$. It is convenient to write consumption in per capita terms: $c(t) = \frac{C(t)}{L^*}$. We assume a CRRA utility function:

$$U(t) = \frac{1}{1-\sigma}c(t)^{1-\sigma} \tag{6}$$

Profits are always invested in physical capital ($\dot{k}(t)$), whereas labour income is optimally allocated between consumption ($c(t)$) and investment in human capital[4] ($\dot{h}(t)$). Therefore:

$$\dot{k}(t) = [1-q(w)]y(h(t),k(t)) \tag{7}$$

$$\dot{h}(t) = q(w)y(h(t),k(t)) - c(t) \tag{8}$$

Households must choose two variables: the wage rate, which defines the capital share and hence the accumulation of physical capital, and the consumption level, which determines the accumulation of human capital.

The Hamiltonian for the problem is:[5]

$$\aleph(c,w,h,k,\lambda,\mu) =$$
$$e^{-\rho t}\tfrac{1}{1-\sigma}c^{1-\sigma} + \lambda[q(w)y(h,k) - c] + \mu[(1-q(w)y(h,k)]$$

The first order conditions are:

$$\aleph'_c = e^{-\rho t}c^{-\sigma} - \lambda = 0 \tag{9}$$

[4]In the text we are implicitly assuming that human capital is specific to each worker, so that each household is interested in investing in it. If human capital were supposed to increase welfare only through externality in the production process, each household should not be interested in investing in education. The traditional free rider problem arises in the presence of externality. In Appendix A we analyse this case, assuming that the Government, through an optimal taxation on labour earnings, push households to invest in education the optimal amount of revenue. We will obtain the same results showed here

[5]In what follows, we do not write the time index unless it is necessary.

$$\aleph'_w = y(h,k)(\lambda - \mu)\frac{dq}{dw} = 0 \tag{10}$$

$$-\aleph'_h = \dot{\lambda} = -[\mu(1-q(w)) + \lambda q(w)]\alpha\frac{y(h,k)}{h} \tag{11}$$

$$-\aleph'_k = \dot{\mu} = -[\mu(1-q(w)) + \lambda q(w)](1-\alpha)\frac{y(h,k)}{k} \tag{12}$$

and transversality conditions are:

$$\lim_{t\to\infty} \lambda(t)h(t) = 0 \tag{13}$$

$$\lim_{t\to\infty} \mu(t)k(t) = 0 \tag{14}$$

Given that $\frac{dq}{dw} \neq 0$, equation 10 implies $\lambda(t) = \mu(t)\forall t$, which, in turn, implies $\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\mu}}{\mu}\forall t$.

The dynamic laws of equations 11 and 12 become, respectively:

$$\frac{\dot{\lambda}}{\lambda} = -\alpha\frac{y}{h} \tag{15}$$

$$\frac{\dot{\mu}}{\mu} = -(1-\alpha)\frac{y}{k}$$

So that:

$$k = \frac{1-\alpha}{\alpha}h \tag{16}$$

Therefore, along the optimal growth path, physical and human capital must grow at the same rate, so that the production function of equation 5 can be written as:

$$y(k) = B\left(\frac{\alpha}{1-\alpha}\right)^{\alpha} k \tag{17}$$

and:

$$y(h) = B\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} h \tag{18}$$

Substituting equation 17 in equation 7, we obtain:

$$\frac{\dot{k}}{k} \equiv g_k = [1-q(w)]B\left(\frac{\alpha}{1-\alpha}\right)^{\alpha} \tag{19}$$

Equation 15 becomes:

$$\frac{\dot{\lambda}}{\lambda} = (1-\alpha)B\left(\frac{\alpha}{1-\alpha}\right)^{\alpha} \tag{20}$$

Substituting equation 18 in equation 8, we obtain the human capital growth rate:

$$\frac{\dot{h}}{h} \equiv g_h = \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} Bq(w) - \frac{c}{h}$$

Given equations 7 and 8, and given the steady state solutions for the per capita product 17 and 18, we can write:

$$\frac{c}{h} = B\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\left(\frac{q(w)}{\alpha} - 1\right) \tag{21}$$

Which implies that, to have a positive consumption, $q(w) > \alpha$ must hold. Note that this condition is implied from the equation 13; in fact, integrating equation 19 and 15 and taking the limit for $t \to \infty$, we obtain $q(w) > \alpha$. Substituting equation 18 in equation 21, we obtain the marginal propensity to consume:

$$\frac{c}{y} = \frac{q(w) - \alpha}{1-\alpha} \tag{22}$$

Differentiating equation 9 with respect to time, we obtain:

$$\frac{\dot{\lambda}}{\lambda} = -\rho - \sigma\frac{\dot{c}}{c}$$

and substituting in equation 20 we obtain the consumption growth rate:

$$\frac{\dot{c}}{c} \equiv g_c = \frac{B\alpha^{\alpha}(1-\alpha)^{1-\alpha} - \rho}{\sigma} \tag{23}$$

The growth rate of consumption is equal to the one of human capital because of equation 21 and because of the constancy of the labour share, and, given equation 16, it must also be equal to the growth rate of physical capital. The per capita production function (equation 5) shows that the optimal economy growth rate (g^*) coincides with the per capita consumption growth rate. Finally, we obtain: $g^* = g_c = g_h = g_k$.

Using equations 23 and 19, we obtain the following definition for the equilibrium labour share:

$$q(w) \equiv q^* = 1 - \frac{1-\alpha}{\sigma}\left[1 - \frac{\rho}{B}\frac{1}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\right] \tag{24}$$

Therefore, the labour share that maximizes expected utility does not depend on wages, but on the "fundamentals", i.e. technology and preferences[6].

Considering equation 23, we can also write the optimal labour share as follow:

$$q^* = \frac{g^*[\sigma - (1-\alpha)] + \rho}{g^*\sigma + \rho}$$

Or, equivalently[7]

$$g^* = \frac{\rho}{\frac{1-\alpha}{1-q} - \sigma}$$

To obtain these results, we supposed that the labour share and the employment rate were constant over time. Obviously, in order to keep the labour share constant, we must have from equation 4:

$$\frac{\dot{w}}{w} = \frac{\dot{v}}{v} \tag{25}$$

From equation 3 we can write the dynamic of the employment:

$$\frac{\dot{L}}{L} = \frac{\dot{A}}{A} - \frac{\dot{v}}{v} \tag{26}$$

Hence, given the definition of A, employment is constant over time if:

$$\frac{\dot{v}}{v} = g^* \tag{27}$$

Equations 25 and 27 imply that, if the labour share is constant, so that equation 25 is respected, employment is constant too and the wage rate grows at the same rate as the whole economy.

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[6]Note that, transversality conditions of equation 14 are filled if $\rho > B(1-\alpha)^{1-\alpha}\alpha^{\alpha}(1-\sigma)$. Therefore the optimal labour share is always higher than the traditional one (the α parameter of the production function), so that $q^* > \alpha$ (with q^* defined in equation 24). Furthermore, the transersality conditions implies a positive consumption (see equation 21) and a positive growth rate of the economic system (see equation 23)

[7]It is straightforward to show that $q^* > \alpha$, so that $c > 0$ implies: $g^* < \frac{\rho}{1-\sigma}$

4 Trade unions, bargaining and optimality

We have no guarantee that bargaining between the social partners, as described in equation 4, leads the economy to the optimal equilibrium of equation 24. To reach the optimal growth path, the bargained wage level should be such that the two equations mentioned are equal, so that:

$$\frac{w^*}{v} = \frac{q^*}{\alpha} \tag{28}$$

where q^* is defined in equation 24. Are there factors which make equation 28 respected?

Let us think of the economy described as a sequence of short-run equilibria, where the trade unions determine the wage rate:

1. considering the existence of a wage rate which maximizes the household utility (rational behaviour)

2. according to their goals, completely ignoring the existence of an optimal wage rate (myopic behaviour);

In the first hypothesis (rational trade unions) the dynamic behaviour of the wage rate is completely determined by considering the optimal wage rate defined in equation 28. Unions operate in order to maximize the expected utility of individuals.

In the second case, unions pursuit their own goal. The standard procedure to solve for the wage in bargaining is based on the Nash bargaining model, which maximizes, with respect to employment and the wage rate, the weighted product of expected gains from bargaining obtained by trade unions and firms. The solution of the model gives the contract curve (equation 3) and the bargained wage rate (w_{SR}), which takes the form:

$$w_{SR} = \frac{\eta + \alpha(1-\eta)}{\eta[1 - z(1-\alpha)] + \alpha(1-\eta)} v$$

where $0 \leq \eta \leq 1$ is the trade unions' bargaining power (and $1-\eta$ is the firms' bargaining power). With efficient bargaining, the labour share of equation 4 becomes:

$$q(\eta) = \alpha + \eta(1-\alpha)$$

The short run labour share as defined in the previous equation and the optimal labour share (q^*) as defined in equation 24 are equal if :

$$\eta^* = \frac{q^* - \alpha}{1 - \alpha} = 1 - \frac{1}{\sigma}\left[1 - \frac{\rho}{B}\frac{1}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\right] \tag{29}$$

which is increasing in σ and ρ and has a maximum for $\alpha = 0.50$.

Therefore, there exists a level of trade-union bargaining power that is "optimal" for the economic system. Consider that unions power (η) in bargaining must coincide with the average propensity to consume as described in equation 22. Empirically, this implies that union power should exceed 50%.

Obviously, on a priori grounds there is no reasons to believe that trade union power is exactly the one described in equation 29. Suppose that bargaining power, and hence wages and the labour share, are lower than optimality: in this case, employment is higher. Even if this situation is a stable steady state, because equation 25 is respected, so that the labour share and the employment rate are constants, the growth rate of human capital is lower than optimality. This is a situation of human capital shortage. Households consume less output, and at a reduced growth rate.

Are there endogenous mechanisms able to lead the bargaining power to the optimal level? It is obvious that trade-unions bargaining power is influenced by various factors, like the unemployment level, the general public's perception that the trade unions are doing the "right thing" in bargaining, the relative level of wages and profits. But, at least in our model, there are no clear factors which can lead the parameter η to the optimal value of equation 29.

Hence, even if unions are indeed useful for economic growth (by raising the wage rate over the competitive one), they should incorporate household optimal behaviour in their objective function.

5 Conclusion

In this paper we have revisited, from a modern perspective, the relationship between the functional distribution of income and growth envisaged by the Ricardian tradition. In an endogenous growth model based on human capital, we have assumed that the revenue accruing to human capital must be split between the social partners according to some bargaining rule, and that the functional distribution of income influences investment in the accumulable factors.

In the short run, efficient bargaining between the social partners determines the employment level and the labour share as a function of the wage rate.

In the long run, the capital share accruing to firms defines the growth path of physical capital, whereas the labour share accruing to households is optimally split between human capital investment and consumption. Hence, households decide the path of human capital accumulation by choosing the amount of income to invest in education.

We have thus obtained analytical results for short run equilibrium (as the outcome of bargaining between firms and unions) and for long run optimal growth (as the outcome of households' intertemporal maximization).

Our main result is that it exists a given labour share depending on preferences and technology alone, which maximizes the expected households utility. This labour share is higher than the "traditional" one, requiring a functional distribution of income more favorable to workers than the one arising from a competitive market ($q^* > \alpha$).

This result depend on the hypothesis of imperfect capital market (not modelled in this paper), which push firms to invest completely their profit in physical capital because of liquidity constraints and leaves the financing of human capital to households alone .

The optimal labour share may be reached thanks to trade union. In fact, our results show that trade unions are required to allow the economic system to reach the optimality, but also that their presence, even is necessary, is not sufficient.

The effective behavior of trade unions leads to the households' desired labour share, only if unions are able to internalize the effect of their behaviour on growth (the "rational" trade unions). Otherwise ("myopic" unions), it exists a given positive bargaining unions power (coinciding with the propensity to consume) which maximize the economy growth rate. The opened question relates to the economic mechanism which may eventually lead the trade unions bargaining power toward the optimal one.

Appendix A: Human capital as externality

We assume that the Government behaves in order to maximize the expected utility of the representative household. Therefore, it withdraws taxes from households and uses the amount it gets to finance the general education system, which must be attended by every individuals. By this way, it is able to solve the coordination problem between individuals arisen by the externality generated by education.

Given the production function (equation 5) and the utility function (equation 6) presented in the text, we must now change the dynamic constraint of equation 8:

$$\dot{h} = \tau q(w) y(h(t), k(t))$$

where τ is the tax rate, whereas the constraint 7 remains unchanged.

The per capita consumption is simply given by:

$$c(t) = (1 - \tau) q(w) y(h(t), k(t))$$

Now, households choose the wage rate (and so the labour share) whereas the Government chooses the rate of taxation. Consequently, firsts order conditions 9 and 10 become, respectively:

$$\aleph_\tau = \left[-e^{-\rho t} [(1 - \tau) q(w) y]^{-\sigma} + \mu \right] qy = 0$$

$$\aleph_w = \left[-e^{-\rho t} [(1 - \tau) q(w) y]^{-\sigma} (1 - \tau) - \lambda + \mu\tau \right] y \frac{dq}{dw} = 0$$

Combining the two first order conditions, we obtain easily $\lambda(t) = \mu(t) \forall t$. Given this result, all the other optimality conditions are completely equals to the one displayed in the text referring to the decentralized solution.

Therefore, with the some procedure shown in the text, we obtain the following results:

- the transersality condition $q(w) > \alpha$ holds;
- the optimal economy growth rate is the some as the one displayed in equation 23;
- the optimal labour share is the some as the one displayed in equation 24; it may also be written as a function of the optimal tax rate:

$$q^* = \frac{\alpha}{\alpha + \tau^*(1 - \alpha)}$$

where the labour share is defined as:

$$\tau^* = \frac{\alpha}{1-\alpha} \frac{\frac{1-\alpha}{\sigma}\left[1 - \frac{\rho}{B}\frac{1}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\right]}{1 - \frac{1-\alpha}{\sigma}\left[1 - \frac{\rho}{B}\frac{1}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\right]}$$

- all the other results presented in the text hold.

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