Growth and distribution. A return to the classical tradition

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Abstract

This article examines the relationship between the functional distribution of income and growth, working with the Goodwin's (1967) model. It develops the idea that capitalists determine employment through their investment policies, and workers choose the distribution of income through a social conflict in the labour market. If this approach is transferred to a setting of long term optimization, interesting results are obtained. If the social classes cooperate in order to maximize the collective well-being, the optimum growth rate comes to depend on the average productivity of capital, on the intertemporal elasticity of substitution, and on the discount rate, as in Rebelo's (1991) model of endogenous growth and the optimal distribution is such to verify the correspondence with the natural rate of growth. When the problem of optimal growth is posed in conditions of conflict between the social classes, then the growth rate approaches more closely to the social optimum, the lesser are the differences between the classes, and the lower the level of conflict between them. Finally, the micro-foundation of saving decisions cannot be complete, because only the ratio between the two propensities to save is determined; but it causes a progressive loss of identity by the social classes until they become indistinguishable.

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1. Introduction

During the Sixties the relation between income distribution and economic growth was at the centre of an interesting debate which involved, amongst others, Kaldor (1955, 1957, 1963), Pasinetti (1962) and Samuelson (1962). Attention focused on the different propensities to save of workers and capitalists, and on the change that occurs in the average rate of saving with variation in the proportions of total income accruing to one or other of these two classes. According to the classical economists the accumulation of capital was the only true engine of economic development, for it "*leads to an improvement in the productive capacity of labour*" (A. Smith, 1776, p. 437). But the problem was that capital can only be increased by saving: "*Capitals are increased by parsimony, and diminished by prodigality and misconduct. Whatever a person saves from his revenue he adds to his capital, and either employs it himself (…) or enables some other person to do so, by lending it to him for an interest*" (A. Smith, 1776, p. 437).

In David Ricardo's 1817 work, *On the Principles of Political Economy and Taxation*, the various pieces of this puzzle are reassembled into a general equilibrium model of growth. As long as profits are positive, the capital stock increases. Given the population, wages will rise above the subsistence level, improving workers' living standards. This will increase the population and the consequent surplus of labour will push wages back down to the subsistence level. Because it is now necessary to produce for a larger population, less fertile land must be brought under cultivation; the distribution of the product from this land will favour the rent, while the profit will diminish. This process will continue until the total output, net of the payment of rent, is just enough to pay subsistence wages to workers; at this point profits are wiped out, the accumulation of capital ceases, and the engine that has driven the growth process is turned of.

In Karl Marx (1887), *Das Kapital: Kritik der Politischen Oekonomie*, edited by F. Engels, the general equilibrium pattern of growth does not differ from Ricardo's model as regards either the main components of the overall structure or the tendency towards long-period steady state due to a fall in the profit rate and cessation of the capital accumulation process. However, Marx was less confident than Ricardo in the ability of technological progress to restore profitability and thus support the accumulation process. He was aware of the instability intrinsic to capitalist economy, and of the inevitable cyclical crisis that would characterize its history as a sequence of booms and slumps until it had been superseded.

The theme of growth treated separately from distribution was taken up again in the first half of the last century by Harrod (1939) and Domar (1946), who developed a dynamic extension of the basic conceptual structures of Keynesian theory: the income multiplier and the role of investments. Both these authors identified the engine of growth in the dual nature of investment, which was simultaneously a component of aggregate demand, and a factor in the expansion of production capacity. If is imposed an equilibrium condition between saving, equal to a constant fraction of income, and the investment necessary to maintain a constant ratio between output and the capital stock, a dynamic equation is obtained which yields the warranted growth rate g. The higher the propensity to save, and the lower the ratio of capital to output, the greater the growth rate will be. The warranted rate *g* represents the maximum rate of growth able to preserve the equilibrium between aggregate demand and supply in a closed economy without intervention by the government. However, it cannot be taken for granted that the equilibrium growth path can be achieved and followed. When the available labour force is fully employed, the economy's growth potential will be equal to the sum of rate of change of the labour force and the rate of Harrod-neutral technical progress. In fact, this rate g* is termed *natural rate of growth.* In the analyses of these two authors, the propensity to save, the capital/output ratio, the population growth rate, and the rate of technical progress, are considered to be constant and exogenous. Consequently, there is no self-regulating endogenous mechanism able to ensure balanced growth with full employment. In general, the situation will instead be $g^* \neq g$, so that the Keynesian analysis of growth cannot be considered as a model of general equilibrium; on the contrary, it describes disequilibrium conditions of opposite sign: an economy that tends to be inflationary when $g>g^*$, or one that is deflationary when $g<g^*$.

Post-Keynesian theory saw an original attempt to fit growth theory within a general equilibrium model able to reconcile the natural and warranted rates of growth. The initial idea was put forward by Nicholas Kaldor (1957), who imagined "*…an economy in which the mechanism of profit and income generation will create sufficient saving ... to balance the investment which entrepreneurs decide to undertake*" (Kaldor and Mirrlees, 1962, p. 344). Total income can be decomposed into the sum of profits and wages. If the propensities to save on the two types of income – from work and capital – are different: $0\leq s_w\leq s_p \leq 1$, then it is possible to define the average propensity to save in terms of the functional distribution of income as follows: $s = s_w + (s_p-s_w)P/Y$. Assuming that the production technology is at fixed technical coefficients, as in Harrod's model, it follows that P/Y = r/a, where r denotes the economy's uniform profit rate and (1/a) the constant capital/output rate. Combining the various components of the model yields the following relation for balanced growth with full employment: $g^* = a[s_w + (s_p-s_w)r/a]$. In the conditions described by the classical economists (where $s_w=0$, and $s_p=1$), the previous 'golden age' assumes the simple form: $g^* = r^*$, known as the 'golden rule' of saving. According to this rule, the saving rate must be equal to the profit rate, which in its turn must be equal to the natural rate of growth.

Recently, attempts to explain the relation between unemployment and growth, and assessing a role of income distribution are proliferating within the classical, Keynesian and post-Keynesian traditions, considering institutional characteristics of the labour market.[1] In fact, in the countries of the European Union, unlike the United States, the

[1] Boyer (1988, 1997). Regulation theory uses the expression 'mode of regulation' to denote the set of norms and institutions forms which comprise both economic and extra-economic dimensions. See two books edited by Salvadori (2003a,2003b), and numerous others books and articles: Alesina and Rodrik (1991,1994) Person and Tabellini (1990, 1992), Bertola (!990, 1993, 1994, 1996), Eicher (1996), De Groot (2001), Bean and Craft (1995),Layard and Nickel (1990), Nickel (1997), Nickel and Layard (1997), Wapler (2001) Addison and Hirsch (1989), Grout (1984), Parreno and Sanchez-Losado (1999) , Palokangus (2003),

strength of the unions seems to have curbed wage inequalities, but it has also slowed the pace of growth: unemployment hampers growth and a slow pace of growth exacerbates unemployment. The effect on growth depends on how aggregate saving is influenced by the distribution of income among the production factors. If (as sustained by Bertola (1993, 1996), unlike by Daveri and Tabellini (1997)), the propensity to save on wages is higher than the propensity to save on profits, then a redistribution of income in favour of profits may foster development, sustaining the accumulation of physical and human capital.

Then, the recent literature on the role of the labour market institutions in economic development highlights that the results depend on the hypotheses adopted on certain basic components. For example, is the marginal propensity to save on wages is greater or is it less than the marginal propensity to save on income from capital? Is the allocation of labour among the sectors rigid or does it respond to marginalist criteria? Choosing one or other hypothesis leads to conclusions that may be even diametrically opposed. However, these questions are no different from those that economists asked prior to the advent of endogenous growth theory[2], when they returned to the Ricardian classical tradition to identify capital accumulation (both physical and human) as the engine of growth and tied the intensity of accumulation to the distribution of income among the social classes – the distinctive component of which was the greater or less capacity to save.

Here I intend to re-examine the relation between the functional distribution of income and growth in the light of classical tradition as re-elaborated in 1967 by R. Goodwin, according to whom profit and wage shares and economic growth are determined by the endogenous solution of the conflict between the social classes. In each period, the capitalists determine the employment level through their saving-investment decisions, and the workers choose the distribution of income, fuelling a degree of conflict coherent with the conditions of the labour market (institutions and rules), which are largely synthesised by the unemployment rate. Then the distribution of income wholly determines the dynamic of the economy, which is characterized by fluctuations of constant amplitude and length around the natural rate of growth, with unemployment rate and profits share cyclically variables in time. The growth cycle ensues substantially from the exogeneity of the propensities to save, which are hypothesised (as in the Ricardian and Marxian tradition) as being respectively nil on wages and one on profits.

However, in a time horizon extended to embrace the entire 'future', it must be admitted that economic agents (workers and capitalists) may decide to allocate their income optimally between saving and consumption, in order to maximize the current value of the utility[3] that they extract from present and future consumption flows.

Rowthorn (1996,1999), Acemoglou (1995,1996,2000), Aghion and Howitt (1992,1994,1998), Wigger (1999), Gilles (1998), Irmen and Wigger (2001), Lingens (2002)

[2] See amongst others , Romer (1986, 1990), Rebelo (1991), Lucas (1988), Barro (1990).

[3] "…. They (endogenous growth models, ndr.) are all long run equilibrium models in which agents are motivated by the lure of profits and the rational search for higher utility and which a market economy

Then, in this article I intend reconstruct the dynamics of different economic systems in relation to two institutional and/or behavioural aspects of fundamental importance in the capitalist economies:

(a) the existence (as in the classical tradition), or alternatively the absence (as in the neoclassical theory) of antagonistic social classes which compete for a more favourable distribution of income;

(b) the propensities to save exogenous (as in the classical tradition) or, alternatively endogenous, decided in order to maximize the current value of flows of future utilities (as in the neoclassical optimal growth theory).

The first aspect to be verified is whether, along the trajectory of socially optimal growth, the cyclicity inherent to the structural form of Goodwin's model are transmitted to the growth rate or whether they remain confined to the employment and to the distribution of income. A second aspect to ascertain is whether, when growth depends on conflict between the social classes, the growth rate thus determined is greater or less than the socially optimal one, and whether it conserves or loses the cyclical component generated by class conflict. Finally, it is of interest to determine whether the micro-foundations of saving decisions does not tend to attenuate the separation between the social classes, rendering them indistinguishable in terms of their capacities to save and accumulate: that is, put in more explicit terms, whether the endogenous saving does logically and rationally entail the disappearance of the social classes.

2. *A model of cyclical growth*

In his celebrate essay *A Growth Cycle* (1967),[4] Richard Goodwin sought to give a precise formal structure to Marx's idea that the alternating phases of expansion and recession, that have characterized capitalism since its beginnings, can be explained by the dynamic interaction among profits, wages and employment.

Goodwin's thesis was that the constitutive structure of capitalist societies is a *homeostatic mechanism* which works through variations in the distributive shares. If real wages increase, profits fall; but when profits fall, saving formation and investments are hampered; as a consequence, new jobs creation stagnates. Because the labour force constantly grows due to increases of both the population and the labour productivity, unemployment tends to increase. The bargaining power of the trade unions is weakened and real wage rises lag behind the growth of labour productivity. Profits revive and stimuli to accumulate capital become increasingly robust. The system enters a new phase of expansion and wages begin to rise again with greater rapidity, in a constant sequence of booms and slumps.

mediates between these self-interested strivings by making them capable of implementation" (Hahn, 1994, p.11)

[4] Other works which have run and extended Goodwin's model are Hoel (1978), Pohjola (1984), Balducci, Candela and Ricci (1984).

This model is an outstanding attempt to incorporate the income distribution – which is necessarily conflictual – into an analysis of the growth of the capitalist economies. It is able to yield interesting results: most notably, an explanation for the possible existence of an underemployment equilibrium as well as for the variability of the distributive shares observable in the short period, and their substantial stability in the long one – what J. Robinson called "*the mystery of constant relative shares*". But also well known are the shortcomings of Goodwin's analysis, and the not entirely convincing elements that it contains: an overly mechanistic description of the workings of complex antagonistic economic forces; an *ad hoc* formulation of the Phillips curve, in which the connection between wage dynamic and employment rate is established in real terms; and an implicit acceptance of Say's law. Nevertheless, Goodwin's model still has indubitable validity and heuristic capacity, given that the profound changes which have taken place in the structure of the social classes have eliminated neither the conflict between them nor their objective complementarity, which constitutes a typical form of symbiosis in economic life.

A description of Goodwin's model will now be provided. All the variables listed below are expressed in *real terms*:

Y aggregate output, by definition equal to total income,
K aggregate capital stock,[5]
a output/capital ratio, by hypothesis constant and exogenous, a=Y/K,
L employment,
b labour productivity, b=Y/L,
β rate of growth of labour productivity, constant and exogenous,
N labour force supply, N(0)=1; for simplicity, the rate of growth is n=0,
e employment rate, *e = L/N* = Y/b,
w wage,
q share of wages in total income, *q = wL/Y= w/b*
s_c propensity to save on profits,
s_w propensity to save on wages,
ρ intertemporal discount rate,
$c_i = (1-s_i)Y_i$ consumption of the i-th social class, i =w,c ,

$U(c_i) = \frac{c_i^{1-s}}{1-s}$ CRRA (constant relative risk aversion) utility function,
è= ñ – (1-ó) β > 0,
γ= è + β = ρ + σβ>0.

[5] For the purposes of the analysis conducted later, the aggregate capital stock can be considered to be capital composed, in fixed proportions, of physical capital and human capital, as hypothesised by Rebelo (1991).

Let us define the *state* variables, i.e. the employment rate and the share of wages in total income:

$$e(t) = L(t) = \frac{Y(t)}{b(t)}$$

$$q(t) = \frac{w(t)L(t)}{Y(t)} = \frac{w(t)}{b(t)}$$

The formal structure of the model consists of two dynamic relations concerning, respectively, the employment rate and the share of wages in income:

(1) $$\dot{e}(t) = e(t)\left(\frac{\dot{Y}(t)}{Y(t)} - \boldsymbol{\beta}\right)$$

(2) $$\dot{q}(t) = q(t)\left(\frac{\dot{w}(t)}{w(t)} - \boldsymbol{\beta}\right)$$

and in the following behavioural hypotheses:

(a) the percentages of saving on profits s_c and on wages s_w are exogenous and constant, and result from customary behaviour, $0 \le s_w \le s_c \le 1$;

(b) saving is entirely invested, so that in the absence of the public sector and the rest of the world, at every instant there is equilibrium between aggregate demand and supply;

(c) the ratio between output and capital a is exogenous and constant; moreover, for the sake of simplicity it is assumed that the capital decay rate is nil. Consequently, the net increase of the capital stock is equal to investment, so that the following relation holds

(3) $$\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{K}(t)}{K(t)} = \frac{I(t)}{K(t)} = a\frac{I(t)}{Y(t)} = a\left[s_c - q(t)(s_c - s_w)\right]$$

(d) with full employment, the real wage grows at a constant rate $(\lambda - \delta) \geq \beta$ above the rate of growth of labour productivity; it diminishes in proportion to the employment rate according to the following function:

(4) $$\frac{\dot{w}(t)}{w(t)} = -\delta + \lambda e(t)$$

which represents a linear Phillips curve expressed in function of the real wage.

By substituting the behavioural equations in the two defining relations, one obtains the structural form of the model, which consists of two non-linear differential equations in the state variables *e(t)* and *q(t):*

(5) $$\dot{e}(t) = e(t)\left[a\left(s_c - q(t)(s_c - s_w)\right) - \mathbf{b}\right]$$

(6) $$\dot{q}(t) = \mathbf{l}q(t)\left[e(t) - \frac{\mathbf{d}+\mathbf{b}}{\mathbf{l}}\right] = \mathbf{l}q(t)(e(t) - e^*)$$

which describes orbits of constant amplitude and periodicity around the steady state equilibrium (q*,e*):[6]

(7) $\dot{e}(t) = 0 \Rightarrow q^* = \dfrac{s_c - \dfrac{b}{a}}{s_c - s_w} < 1$ on condition that: $s_w < \beta/a < s_c$

(8) $\dot{q}(t) = 0 \Rightarrow e^* = \dfrac{\mathbf{d}+\mathbf{b}}{\mathbf{l}} \leq 1$ if $(\mathbf{l}\text{-}\mathbf{d}) \geq \beta$

3. Multiplicity of dynamic equilibria

If we take the model set out in section 2 as the basis for analysis, we can reconstruct the dynamics of different economic systems in relation to two institutional and/or behavioural aspects of fundamental importance in the capitalist economies:

(c) the existence, or alternatively the absence of antagonistic social classes which compete for a more favourable distribution of income;

(d) the propensities to save exogenous or, alternatively endogenous, decided in order to maximize the current value of flows of future utilities.

The cases generated by the combination of these various structural features are shown in Table 1 below:

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[6] There is another steady state condition: e*=0, q*=0, which requires a nil wages share and a nil employment rate. This is evidently not a vital equilibrium, and we may ignore it. Moreover, given e*, it is easy to determine the steady state path of the income: $Y^*(t) = e^* e^{bt}$.

TABLE 1: ***Taxonomy of dynamic equilibria***

Saving decisions	**Society without social classes**	**Society with social classes**
Exogenous	**Case A** Harrod model Unstable equilibrium	**Case C** Goodwin model Stable cyclical equilibrium
Endogenous	**Case B** Optimal growth Stable cooperative equilibrium	**Case D** Sub-optimal growth Unstable non-cooperative equilibrium

We may begin with analysis of the possible equilibria generated in case C.

3.1. Case C: *society with social classes and exogenous saving.*

It is evident that the base model illustrated above pertains to **case C**; in the economy there are two antagonistic social classes which behave in customary manner in their saving and investment decisions, reproducing the celebrated model set out in Goodwin's *A Growth Cycle*.

According to this model, the distribution of income is conflictual and wholly determines the dynamic of the economic system, which is characterized by fluctuations of constant amplitude and length around the natural growth rate β with the employment rate and the wages share varying in time[7]. Therefore, the equilibrium rate of growth of income is:

$$\frac{\dot{e}(t)}{e(t)} = 0 \Rightarrow g_C = \frac{\dot{Y}(t)}{Y(t)} = \beta$$

while the steady-state share of income from work depends entirely on the gap between the exogenous propensities to save of the two social classes:

$$q_C{}^* = \frac{s_c - \beta / a}{s_c - s_w} < 1$$ on condition that : $s_w < \beta/a < s_c$

[7] For a demonstration of this result, see Balducci and Candela (1982), pp.45-49.

Note that *the condition* $s_c > s_w$ *, i.e. the existence of a gap between the propensities to save of the two social classes, is fundamental in fuelling the antagonistic process that drives the capitalist economy.*

As we can see, all that is important relative to the long-period phenomena like growth and income distribution is entirely defined by the saving decisions of capitalists and workers. Workers are able to control only the amplitude of the fluctuations by regulating their maximum wage demands or the intensity of their reaction to changes in employment in equation (4). The greater the value of ä, the lower, *ceteris paribus*, the value of q* and the less marked the fluctuations.

I now consider the remaining cases described in the taxonomic table 1.

3.2. Case A: *society without social classes and exogenous saving*

These features are immediately recognizable as the fundamental components of Harrod's model of exogenous growth. Indeed, setting $s_c = s_w = s$, in equation (5) the equilibrium rate of growth of income is entirely determined by the propensity to save (s) and by the technological parameter (a):

(9a) $$\frac{\dot{e}(t)}{e(t)} = as - \boldsymbol{b} \Rightarrow g_A = \frac{\dot{Y}(t)}{Y(t)} = as$$

(9b) q_A is undetermined

Note that *e(t)* represents the employment rate, which can at most reach unitary value, i.e. full employment, beyond which point it can rise no further. Hence, in the long period, saving and investment decisions cannot be "*entirely exogenous*"; rather, it must be: $s^* = \boldsymbol{b}/a$. However, because there are no endogenous mechanisms able to 'persuade' the economy to save exactly the share (β/a) of its income, the model is structurally unstable.

Only public intervention with suitable taxation or subsidies for saving and investment can ensure growth at the constant natural rate *g*. If the government levies taxes at proportional rate *ô* and invests the whole yield, equation (9a) becomes: $as(1-\boldsymbol{t})+a\boldsymbol{t} = \boldsymbol{b}$, and the stability of the full employment equilibrium will be ensured by the tax rate:

$$\boldsymbol{t}^* = \frac{\frac{\boldsymbol{b}}{a} - s}{1-s} \leq 1$$

Preliminary comparison of the effects induced by social conflict is now possible. Obviously, the steady-state growth rate of income is the same in both cases, which implies – on average – the same saving rate ($\boldsymbol{b}/a$) made possible: (i) in Goodwin's model by a particular distribution of income which, in its turn, is closely dependent on the gap between the propensities to save of the two social classes; and (ii) in Harrod's model by an appropriate fiscal policy.

3.3. **Case B.** *Economy without (conflict between) social classes and optimal saving*

If we shift our attention away from cycle and examine the long-period phenomena of economic growth by extending the time horizon to infinity, it is not possible to consider the saving completely exogenous, founded on immutable habits. One must almost admit that economic agents select the optimal saving in order to maximize the current value of the utility that they extract from consumption. In other words, the propensity to save should be addressed as a problem of intertemporal optimization.

Then, we may investigate the optimal growth trajectory of an economy without conflict between the social classes on the basis of Goodwin's model constituted by equations (5) and (6), and assuming that workers and capitalist decide to cooperate in order to maximize the weighted sum (the weights being respectively φ[8] and (1- φ)) of their utilities by choosing the propensities to save s_c and s_w.

Assuming identical CRRA utility functions, we may write the social utility function as follows:

$$U(c(t)) = \varphi U(c_w(t)) + (1-\varphi)U(c_c(t)) = e^{b(1-\sigma)t}\frac{e(t)^{1-\sigma}}{1-\sigma}\left[\varphi\left((1-s_w)q\right)^{1-\sigma} + (1-\varphi)\left((1-s_c)(1-q)\right)^{1-\sigma}\right]$$

The Hamiltonian function therefore takes the following form:

$$H(t) = e^{-qt}U(c(t)) + \mu_1(t)e(t)\left[a\left(s_c - q(t)(s_c - s_w)\right) - \beta\right] + \mu_2(t)\,\lambda q(t)\left[e(t) - e^*\right]$$

where μ_1 (t) and μ_2 (t) are the co-state variables.

The first-order maximum conditions are the following:

(11a) $$\varphi e^{-qt}e(t)^{-\sigma}\left((1-s_w)q(t)\right)^{-\sigma} = a\mu_1(t)$$

(11b) $$(1-\varphi)e^{-qt}e(t)^{-\sigma}\left((1-s_c)(1-q(t))\right)^{-\sigma} = a\mu_1(t)$$

[8] The weights φ and (1-φ) can represent the numerousness of the two social classes.

Taking account of conditions (11a) and (11b), and simplifying in appropriate manner, we obtain the dynamic laws of the co-state variables and the limit conditions:

(12) $$\dot{m}_1(t) = -m_1(t)(a - \mathbf{b}) - m_2(t)\,\mathbf{l}q(t)$$

(13) $$\dot{m}_2(t) = -m_2(t)\,\mathbf{l}(e(t) - e^*)$$

(14a) $$\lim_{t \to \infty} m_1(t)e(t) = 0$$

(14b) $$\lim_{t \to \infty} m_2(t)q(t) = 0$$

Moreover, the equality of the equations (11a) and (11b) yields the following relation between wages share and propensities to save:

(15) $$q_B = \frac{1}{1 + \frac{1 - s_w}{1 - s_c}\left(\frac{1 - \mathbf{j}}{\mathbf{j}}\right)^{\frac{1}{s}}}$$

which, on attributing for simplicity equal weights to the two classes express the current value, φ=(1-φ), straightforwardly becomes:

(15a) $q_B = \dfrac{1 - s_c}{2 - s_c - s_w}$ or alternatively:

(15b) $(1-s_w)q_B = (1-s_c)(1-q_B)$

and states that in steady state equilibrium, consumption by the two social classes must be equal because they obtain the same marginal utility from it.

Using equations (13) and (6) to develop the limit condition (14b), it can be easily shown that the following convergence condition must hold: ì$_2$(t)=0, given that q(0) is usually not nil: hence, ì$_2$(t)=0 for every t. Consequently, (12) becomes:

(12a) $$-\frac{\dot{m}_1(t)}{m_1(t)} = a - \mathbf{b}$$

Taking the *ln* of (11a) (or equally of (11b)), differentiating with respect to time (bearing in mind that in steady state equilibrium the propensities to save are constant and that the share *q(t)* is also constant for optimum choice (implied by the condition ì2(t)=0), and inserting (12a), we obtain the following optimum rate of growth of the income:

$$(16)\qquad \frac{\dot{e}(t)}{e(t)} = \frac{1}{s}\left[a - g\right] \Rightarrow g_B = \frac{\dot{Y}(t)}{Y(t)} = \frac{a - r}{s}$$

Moreover, on inserting equations (16) and (12a) in the limit condition (14a), we find that this entails the following relation between parameters, which represents both a condition for steady state convergence, and a condition for the economy's viability:

è = ñ-(1-ó)β >0 and, therefore: a>ñ>(1-ó) β or also: a> γ>β

By comparing (16) with (5), we can determine the equilibrium average propensity to save in function of the fundamental technological *(a*) and behavioural (*ñ, ó*) parameters:

$$s^{B}_{mean} = \frac{a - r}{a s}$$

Finally, it is easy to verify from condition (15) that the contribution to saving of the two classes must be proportional to the relative weight of each of them, and must therefore be equal when the two classes are weighted in the same way. Consequently, even if the propensities to save were identical: $s_W = s_C =: s_{mean}$ total income would be divided exactly in half between the two social classes.

In any case, given the average propensity s_{mean} and one of the two propensities to save (s_W or s_C), there is a single distribution of income able to ensure the economy's stability along the balanced growth path of full employment. In effect, we have two equations: the (15b), which defines the distributive share in function of the propensity to save, and the equation which defines the average propensity to save:(qs_W+ (1-q)s_C)= (a-ñ)/aó , while the variables to determine are three: the labour share *q** and the two propensities to save s_W and s_C. The model can endogenously determine only the distributive share and the relative ratio between the two propensities and, therefore, the micro-foundation of the saving cannot be complete. Closure of the model requires that one of the two propensities is exogenous. Given s_W, it is possible to determine explicitly both the wages share q(s_W) and the propensity to save s_C(s_W):

$$(17)\qquad q^{*}_{B}(s_w) = \frac{1 - \frac{1}{s}\left(1 - \frac{r}{a}\right)}{2(1 - s_w)}$$

$$(18)\qquad s_c(s_w) = \frac{\left(1 - \frac{r}{a}\right)(2 - s_w) - s s_w}{\left(1 - \frac{r}{a}\right) + s(1 - 2s_w)}$$

Values of the parameters: a=0.1; β*=0.02;* σ*=0.9;* ρ*=0.04*

Figure 1: relation $s_c(s_w)$

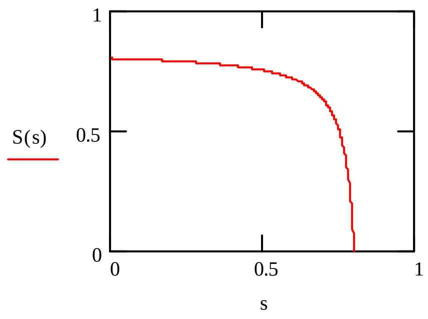
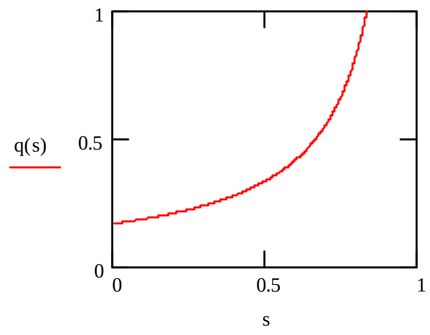


Figure 2: labour share $q(s_w)$



3.4. **Case D**: *society with conflict between the social classes and optimum saving.*

If there are two agents interacting in the economic *arena,* then the optimum problems that we must deal with are two, one for each agent. Moreover, we must define the concept of equilibrium. For our purposes here, it seems correct to maintain that each social class defines its optimum saving by taking the antagonist class's strategy as given and behaving in perfectly symmetrical manner. The equilibrium will therefore be defined at the intersection between the two optimal strategies as a Nash non-cooperative equilibrium.

This model can be solved by introducing the Hamiltonians of the two intertemporal optimum problems, the first for the class of workers, and the second for the class of capitalists:[9]

$$H(w) = e^{-qt}\frac{c_w(t)^{1-s}}{1-\sigma} + \mu_1(t)\left[a(s_c - q(t)(s_c - s_w)) - \beta\right] + \mu_2(t)\,\lambda q(t)(e(t) - e^*)$$

$$H(c) = e^{-qt}\frac{c_c(t)^{1-s}}{1-\sigma} + \eta_1(t)\left[a(s_c - q(t)(s_c - s_w)) - \beta\right] + \eta_2(t)\,\lambda q(t)(e(t) - e^*)$$

[9] It is hypothesised that the two social classes have the same discount rate and the same parameters for the utility function because the concern of this paper is to examine other features distinguishing the social classes.

The first-order maximum conditions are the following (where a denotes the conditions regarding the workers and b those regarding the capitalists):[10]

(19a) $$e^{-qt} c_W(t)^{-s} = am_1(t)$$

(19b) $$e^{-qt} c_C(t)^{-s} = a\boldsymbol{h}_1(t)$$

(20a) $$-\frac{\dot{m}_1(t)}{m_1(t)} = a\left[s_c + q(t)(1-s_c)\right] - \boldsymbol{b} + \boldsymbol{l}q(t)\frac{m_2(t)}{m_1(t)}$$

(20b) $$-\frac{\dot{\boldsymbol{h}}_1(t)}{\boldsymbol{h}_1(t)} = a\left[1 - q(t)(1-s_w)\right] - \boldsymbol{b} + \boldsymbol{l}q(t)\frac{\boldsymbol{h}_2(t)}{\boldsymbol{h}_1(t)}$$

(21a) $$-\frac{\dot{m}_2(t)}{m_2(t)} = ae(t)(1-s_c)\frac{m_1(t)}{m_2(t)} + \boldsymbol{l}(e(t) - e^*)$$

(21b) $$-\frac{\dot{\boldsymbol{h}}_2(t)}{\boldsymbol{h}_2(t)} = -ae(t)(1-s_w)\frac{\boldsymbol{h}_1(t)}{\boldsymbol{h}_2(t)} + \boldsymbol{l}(e(t) - e^*)$$

Though omitting for the sake of brevity the four limit conditions, one for each state variable and for each optimum problem, we must nevertheless add the conditions for intersection between the reaction functions of each social class. These can be written as equality between the rates of variation of the costate variables which – as well known – measure variations in the respective state variables *e(t)* and *q(t)*:

(22a) $$\frac{\dot{m}_1(t)}{m_1(t)} = \frac{\dot{\boldsymbol{h}}_1(t)}{\boldsymbol{h}_1(t)}$$

(22b) $$\frac{\dot{m}_2(t)}{m_2(t)} = \frac{\dot{\boldsymbol{h}}_2(t)}{\boldsymbol{h}_2(t)}$$

[10] Note that the only difference between the first-order conditions (19a, b) and the analogous conditions (11a, b) is that the evaluations of capital accumulation in terms of marginal utility of consumption are in this case different for the two social classes, while in the cooperative case they were identical (the weight attributed to the two social classes remaining equal).

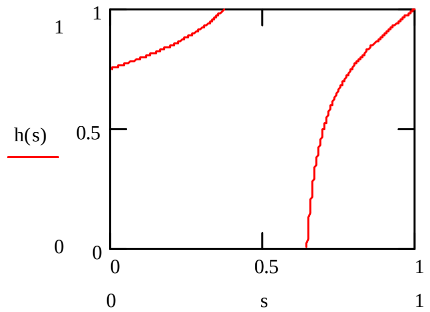
The optimal rate of growth of income in function of the propensities to save of the two social classes is the following (see *Appendix A)* :

(26) $$\frac{\dot{e}(t)}{e(t)} = \frac{1}{\mathbf{s}}\left[ah(s_w, s_c) - \mathbf{g}\right] \Rightarrow g_D = \frac{\dot{Y}(t)}{Y(t)} = \frac{1}{\mathbf{s}}\left[ah(s_c, s_w) - \mathbf{r}\right] \geq 0 \qquad \text{if: } h(s_w, s_c) \geq \frac{\mathbf{r}}{a}$$

where: $h(s_w, s_c) = \dfrac{1 - s_c s_w}{2 - s_c - s_w} \leq 1$

is a function of the propensities to save of the two classes, which is ½ in the case of nil saving: h(0,0)=1/2. This reaches its maximum in the classic case of the maximum difference of the propensities to save: s_W=0, s_C=1: h(0,1)=1; assumes a continuum of intermediate values in the case of uniformity between the propensities to save: s_W=s_C=s: h(s,s)= (1+s)/2 ; and is indeterminate if both propensities are equal to one.

Figure 3 : h(s_C(s_W),s_W)
Values of the parameters: a=0.1; ***b****=0.02;* ***s****=0.9;* ***r****=0.04*



Therefore, given that h(s_C ,s_W)   1 it can be easily ascertained that whatever the values of the propensities to save of the two social classes, the rate of growth in this conflictual economy g_D is less than (or at most equal to) the one that would obtain in a cooperative economy g_B.

Moreover, given that the rate of growth defined by (26) must be equal to the one defined by equation (5), we can define both the average propensity to save:

(27) $$s^D_{mean} = \frac{ah(s_w, s_c) - \mathbf{r}}{a\mathbf{s}} \leq s^B_{mean}$$

and the optimum wages share in function of the average propensity to save:

(28) $$q_D^* = \frac{s_C - s_{mean}^D}{s_C - s_W}$$

It should be noted that the model cannot determine simultaneously all the variables, the growth rate g_D , the distributive share q_D and both the propensities to save, s_c and $s_{w.}$ The reason is that the model is composed by two differential equations (which determine $g_D(s_c,s_w)$ and $q_D(s_c,s_w)$), and two first order conditions, which will determine s_c and s_w . Unluckily, these first order conditions establish that the steady state distributive shares must be constant, but they are not ables to fix an exact value of $q_{D.}$ Therefore, it is necessary to give as an exogenous variable or $q_{D,}$ or one of the two propensity to save.

Given s_w, it is possible to determine explicitly both the propensity to save $s_c(s_w)$ (see figure 4) and the wages share $q_D(s_w)$ (see figure 5):

(29) $$s_c(s_w) = \frac{1 - \frac{r}{a}}{s_w + s(1 - 2s_w)}$$

(30) $$q_D^*(s_w) = \frac{s_C(s_w) - s_{mean}^D(s_c(s_w), s_w)}{s_C(s_w) - s_W}$$

Values of the parameters: a=0.1; ***b****=0.02;* ***s****=0.9;* ***r****=0.04*

Figure 4: relation $s_c(s_w)$

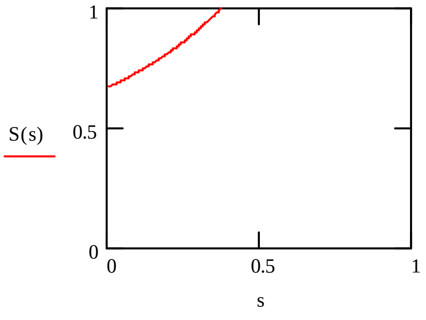


Figure 5: labour share $q(s_w)$

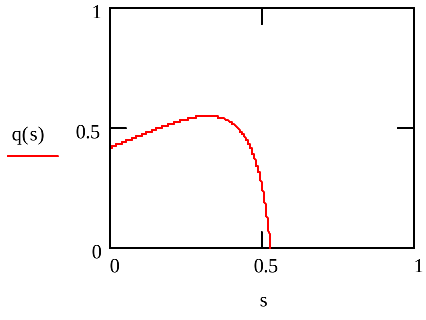
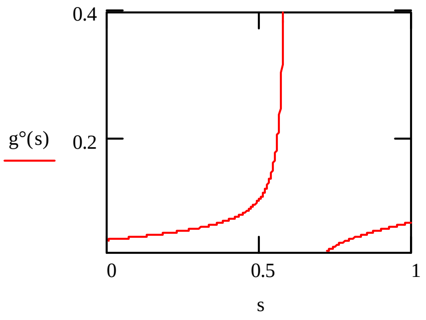


Figure 6: growth rate g(sw)



Finally, on inserting equations (19a) and (26) in the first limit condition: $\lim_{t\to\infty} m_1(t)e(t) = 0$, we obtain the following condition for steady state convergence:

$$h(s_w, s_c) < \frac{\boldsymbol{r}}{a(1-\boldsymbol{s})}$$

It easy to ascertain that, if σ→0 and, or ρ→0, this convergence condition is not verified, because : ½ ≤ $h(s_c,s_w)$ ≤ 1.

Then, although the structural dynamic is that of a stable cycle (this being a feature peculiar to Goodwin’s model), once it has been subjected to the optimal choice of the social classes, it may produce a growth path that can, or cannot converge to the steady state in relation to the degree of risk aversion and, or to the discount rate. In any case the dynamics is not cyclical.

TABLE 2: ***The typology of steady-state equilibria***

Saving decisions	Society without social classes (indeterminate income distribution)	Society with social classes (endogenous income distribution)	Effects of conflict
Exogenous	***Case A:*** *Harrod model* $G_A = as \neq \beta$ Q_A undetermined	**Case C:** *Goodwin cycle* $g_C = \beta$ $q_C = \frac{s_C - \beta / a}{s_C - s_W}$	*Conflict* generates constant cycles around the steady-state equilibrium.
Endogenous	***Case B:*** *cooperative equilibrium* $g_B = \frac{a - \boldsymbol{r}}{\boldsymbol{s}}$ $q_B = \frac{1 - s_c}{2 - s_c - s_w}$	***Caso D:*** *non-cooperative equilibrium* $g_D = \frac{ah(s_c, s_w) - \boldsymbol{r}}{\boldsymbol{s}}$ $q_D = \frac{s_C - s^D_{media}}{s_C - s_W}$	*Conflict* reduces the rate of growth and makes the model unstable
Effects of optimal saving	The *optimal saving decisions* increases the rate of growth of the income if: $s < \frac{a - \boldsymbol{r}}{a\boldsymbol{s}} = s^B_{mean}$	The *optimal saving decisions* increases the rate of growth of the income if: $h(.) > (\rho + \sigma\beta)/a$	

4. Conflict between the social classes and multiplicity of equilibria.

It is evident from equations (26) and (27) that the average propensity to save, and therefore the optimum rate of growth of the income, depends on the solution of the distributive problem, which is typically conflicting. Each of these variables assumes a particular value in function of the value assumed by the function $h(s_c, s_w)$ defined by equation (25), which comprises the strategic variables of the two social classes $0 \le s_w \le 1$, $0 \le s_c \le 1$, and assumes a value normally between ½ and 1, as shown above (see figure 3).

Consequently, the rate of growth will assume a continuum of values ranging from a minimum value when saving is nil and the economy chooses to 'consume', instant by instant, the stock of capital previously accumulated, and a maximum value corresponding to the cooperative solution (see also figure 6):

$$g^D_{\min} = \frac{\frac{1}{2}a - \boldsymbol{r}}{\boldsymbol{s}} \le g_D = \frac{\frac{1+s}{2}a - \boldsymbol{r}}{\boldsymbol{s}} \le g^D_{\max} = \frac{a - \boldsymbol{r}}{\boldsymbol{s}}$$

while intermediate growth rates depend on the values assigned to the propensity to save by each social class.[11]

It is possible, however, to restrict the number of acceptable equilibria by observing that it is not admissible that the two propensities should simultaneously assume unitary value, and that – usually – capitalists have a propensity to save at least equal to that of workers. We may therefore introduce the following useful restrictions in the equation (29):

$$0 \le s_W < 1, \qquad 0 < s_C \le 1 , \qquad s_W \le s_C$$

But even if the number of acceptable equilibria is restricted in this way, the problem is still not solved. The reason is that the economy's pace of growth defines – if we wish to use the expression – the size of the *cake* to be divided up; but equally important is the way in which it is divided, i.e. the definition of the distributive shares. In fact, on observing equation (28) one realizes that the greater the average propensity to save, the more rapid is the rate of growth, but – *ceteris paribus* – the lower the share of income that goes to labour, and vice versa (*see figure 5*).

Therefore, from the workers' point of view, the best solution would be the one put forward in the classical literature: $s_W = 0$, $s_C = 1$, i.e. the capitalists entirely save and the workers entirely consume their respective incomes, engaging in a typical form of free riding. In this case, h(0,1)=1, the growth rate would be the maximum possible, equal to the social optimum one, and the wages share would be as follows:

$$q^D_{13} = 1 - \frac{a - \boldsymbol{r}}{a\boldsymbol{s}} .$$

It is equally evident that the best solution for the capitalists would – rather paradoxically – be the exact opposite: $s_W=1$, $s_C=0$, which we must exclude, however, in

[11] In order to have definite and precise solutions of the propensities to save of the two social classes, it would be necessary to include significant differences either in the discount rate or in the parameters of the respective utility functions – which was excluded in note 35 for the reasons then given

order to maintain a *minimum of coherence* with the definition of social class in the economic sense (where capitalists are the owners of capital: a definition impossible to justify if they did not save at all!).

Mention of the paradoxical situation of capitalists who do not save serves to highlight that – following the workers' free riding behaviour – it is also in the capitalists' interest to adjust by reducing to zero their propensity to save. In this case, we have h(0,0)=1/2 and a minimum or negative growth rate as a typical Nash non-cooperative equilibrium which disadvantages both classes.

Before proceeding further with the discussion, it is advisable to arrange the possible cases in a table by combining the acceptable strategies of the two social classes.

TABLE 3: ***the multiplicity of non-cooperative equilibria***

	$s_C=0$	$s_C=s<1$	$s_C=1$[12]
$s_w = 0$	**Case D11** $g_D^{11} = \frac{1}{s}(\frac{a}{2} - r)$ $q_D^{11} \Rightarrow ind.$	**Case D12** $g_D^{12} = \frac{1}{s}(\frac{1}{2-s}a - r)$ $q_D^{12} = 1 - \frac{a - r(2-s)}{as(2-s)s}$	**Case D13** $g_D^{13} = \frac{1}{s}(a - r)$ $q_D^{13} = 1 - \frac{a - r}{as}$
$s_w = s<1$	**Case D21** *Non acceptable*	**Case D22** $g_D^{22} = \frac{1}{s}(\frac{1+s}{2}a - r)$ $q_D^{22} \Rightarrow undetermined$	**Case D23** $g_D^{23} = \frac{1}{s}(a - r)$ $q_D^{23} = \frac{r - a(1-s)}{as(1-s)}$

However, even if the Nash equilibrium seems to be the inevitable 'rational' outcome of a one-off non cooperative game with information symmetry, in reality cannot be a '*true*' game equilibrium, because it determines only the optimum rate of income decrease while failing to resolve the distributive problem. This consideration should convince us of the following two important points:

(i) the Nash equilibrium does not lend itself well to describing the interactions between the social classes which take place over time, generating learning and agreement processes that should restrict free riding and ensure the economy's survival; and

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[12] Note that $s_c=1$ allows the maximum growth rate to be achieved (corresponding to the cooperative one) independently of the workers' propensity to save, which is instead important for defining the distributive share. This result reproduces, in a very different context, that of Pasinetti (1962)

(ii) in the context of the Nash equilibrium, the problems of growth and distribution cannot be solved simultaneously.

In the light of these considerations it is possible to imagine, I think, other types of *'mediation'* or *'implicit agreement'* equilibrium, which can prevent both the self-destructive "*irrationality*" of the prisoner's dilemma ($s_w=s_c=0$), and the unlikely *surplus of collaboration* necessary to sustain the social optimum equilibrium. The idea is that the free riding behaviour will cease to be inevitable when a symmetry of strategies comes about between the social classes.

Let us therefore assume that, at time t=t°, the propensities to save of the two classes are by customary behaviour both positive and different from each other: $s_w(t°)=s_w° > 0$ and $s_c(t°)=s_c°>s_w°$. Let us then suppose that, after that time, each class begins to behave 'rationally' – i.e. as a free rider – reducing its propensity to save. Let us finally assume that the velocity of capitalists' reduction in their propensity to save is greater than that of the workers, so that at time t'>t° it is the case that $s_w(t')=s_c(t')=s(t')>0$. When symmetry of strategies has been reached, it is probable that both social classes will cease to reduce their propensities to save, so that the type D_{22} equilibrium described in Table 3 arises. In fact, however, the *mediation* equilibrium D_{22} entails a progressive loss of identity by the social classes, a consequent decline in social conflict, and the onset of a steady-state *mediation* equilibrium which is sub-optimal with respect to the social optimum equilibrium but preferable to the self-destructive Nash (non-cooperative) one.

5. Conclusions

Along the steady-state growth path, the economy grows at a constant rate equal to the exogenous rate of growth of labour productivity. If, however, the classes cooperate in order to maximize the collective well-being, the social optimum growth rate comes to depend on the average productivity of capital, on the intertemporal elasticity of substitution, and on the discount rate – in like manner to Rebelo's (1991) model of endogenous growth. The optimal distribution of income and the average propensity to save are such to justify the equality of the optimal and natural growth rates. But the economic fluctuations, which are inherent to the Goodwin's model, disappear.

When the problem of optimal growth is faced up in conditions of conflict between the social classes – assuming, that is to say, that capitalists and workers maximize the utility which they extract from their specific consumption – then it is possible to have a multiplicity of the unstable non-cooperative equilibria. However, each of these non-cooperative growth rates approaches more closely to the social optimum one, the less marked are the differences between the classes, and the lower the level of conflict between them.

Finally, the micro-foundation of saving decisions (both in a cooperative context and in conditions of social conflict) cannot be complete, because only the ratio between the two propensities to save is determined; but it causes a progressive loss of identity by the social classes until they become indistinguishable.

Appendix A

We must take the *ln* of equations (19a) and (19b) and differentiate with respect to time, bearing in mind that the propensities to save must be constant in the steady state equilibrium. We thus obtain from (19a) and (19b) respectively the definitions of the rates of variation in consumption per unit of efficiency of each social class:

(19a)’ $$\frac{\dot{c}_w(t)}{c_w(t)} = \frac{\dot{e}(t)}{e(t)} + \frac{\dot{q}(t)}{q(t)} = \frac{1}{s}\left[-\frac{\dot{m}_1(t)}{m_1(t)} - q\right]$$

(19b)’ $$\frac{\dot{c}_c(t)}{c_c(t)} = \frac{\dot{e}(t)}{e(t)} - \frac{\dot{q}(t)}{q(t)} = \frac{1}{s}\left[-\frac{\dot{h}_1(t)}{h_1(t)} - q\right]$$

In equilibrium, because the rate of growth of employment included in the two equations refers to the economy, it must be the same. By contrast, the workers’ income share q(t) will vary in the opposite direction to the capitalists’ share of income (1-q(t)). Therefore, from (19a)’ and (19b)’ and condition (22a) , the following condition is obtained:

(22c) $$\frac{\dot{q}(t)}{q(t)} = -\frac{\dot{q}(t)}{q(t)} = 0$$

i.e. the distributive shares must remain constant in the Nash equilibrium and, consequently, also the rates of growth of consumption by the two classes will be equal in the steady state.

Using equations (20)-(22a,b,c) we may define both the ratio between the costate variables and their rate of variation:

(23a) $$\frac{m_2}{m_1} = \frac{(1-s_c)a}{gq}\left[\frac{1-s_c}{2-s_c-s_w} - q\right]$$

(23b) $$\frac{m_2}{m_1} = -\frac{1-s_c}{1-s_w}\frac{h_2}{h_1}$$

(24) $$-\frac{\dot{m}_1}{m_1} = -\frac{\dot{h}_1}{h_1} = ah(s_w, s_c) - b$$

where we have indicated with:

(25) $$h(s_w, s_c) = \frac{1 - s_c s_w}{2 - s_c - s_w} \leq 1$$

On inserting equations (23a) and (25) in (19a)’, we obtain the optimal rate of growth of income per unit of efficiency in function of the propensities to save of the two social classes, described in the text (*equation (26*)).

Moreover, if we take the *ln* of equations (23b) and differentiate with respect to time, bearing in mind that the propensities to save must be constant in the steady state equilibrium, we obtain the following equality:

(31) $$\frac{\dot{h}_2(t)}{h_2(t)} - \frac{\dot{h}_1(t)}{h_1(t)} = \frac{\dot{m}_2(t)}{m_2(t)} - \frac{\dot{m}_1(t)}{m_1(t)}$$

Using equations (20a), (20b), (21a) and (21b), and simplifying we obtain:

(32) $$e(t)\left[(1 - s_w)\frac{h_1(t)}{h_2(t)} + (1 - s_c)\frac{m_1(t)}{m_2(t)}\right] + \frac{l q(t)}{a}\left[\frac{h_2(t)}{h_1(t)} - \frac{m_2(t)}{m_1(t)}\right] + \left[(1 - s_c) - q(t)(2 - s_c - s_w)\right] = 0$$

It is easy to ascertain that the expression in the first square brackets is equal to zero for the equation (23b), the expression in the second square brackets is equal to zero for the equation (22b); then, the equation (32) is verified only if :

(33) $$q_D = \frac{1 - s_c}{2 - s_c - s_w}$$

Finally, equalizing the equations (28) and (33), we obtain the optimal relation between the two propensities to save (29) written in the text.

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