Are Kaleckian models of growth and distribution relevant for the long run?*

Abstract

In this paper, we compare two alternative post Keynesian approaches to the analysis of long run economic growth. In the Classical approach accumulation is governed by households saving choices and the normal degree of capacity utilisation is exogenous; in the Kaleckian approach capital accumulation is driven by an exogenous component, representing the Keynesian animal spirits and the normal degree of capacity utilisation is endogenous.

We adopt a common framework based on the following assumptions. Firstly, we frame the analysis in discrete time. Secondly, we introduce an explicit mechanism incorporating firms' constant effort to bring together the current and the normal degree of capacity utilisation to take into account the neo Ricardian critique to the Kaleckian approach. Finally, we introduce a simple nonlinearity into the pricing equation.

We also explore the dynamic properties of the Classical and Kaleckian specifications of the basic model. For both specifications, we show that, when the steady growth path is not an attractor, the comparative statics properties of the model do not always hold. Economic growth and profitability could be both demand or supply led depending on the value of the parameters.

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Are Kaleckian models of growth and distribution relevant for the long run?

1. Introduction

In seminal contributions (see for example Rowthorn, 1981; Dutt, 1984, 1990; Nell, 1985; Amadeo 1986a, 1986b, 1987; and Lavoie 1992 and 1995), Kaleckian models of growth and distribution have been proposed in which demand plays a crucial role in the determination of the most important macro-variables (in particular the rate of profits and the capital accumulation rate) via firms' investment decisions.[1] Typically, Kaleckian analyses involve the comparison of steady growth paths in order to study the long term properties of the economy. The two most important results of Kaleckian analyses are the 'paradox of thrift' and the 'paradox of costs'. According to the former, an increase in the propensity to save reduces capital accumulation and the rate of profits: economic growth and profitability are consumption led. According to the latter, an increase in the wage rate, bringing about an increase in the wage share in national income, increases growth and profit rates: economic growth and profitability are wage led. That is, in Kaleckian analyses demand spurs growth also favouring profitability.

However, the standard Kaleckian model has been criticised by neo Ricardian authors for being ill-specified. In particular, it leaves out the adjustment mechanism of the actual to the 'normal' degree of capacity utilisation, that is, the degree of capacity utilisation that, for its technical, conventional or strategic nature, firms expect to prevail (see Committeri, 1986; Kurz, 1986; Garegnani, 1992; Garegnani and Palumbo, 1998, Ciampalini and Vianello, 2000; and for a recent assessment of the

[1] The Kaleckian approach to growth and distribution is inspired by the works of Kalecki (1971) and Steindl (1956).

debate Commendatore *et al.*, 2003). Once such a mechanism is introduced, the results of the standard Kaleckian model do not hold and demand ceases to affect growth in the long run.[2]

Attempts to reconciliation between Kaleckians and neo Ricardians are proposed by Dumenil and Levy (1999), Chick and Caserta (1997), Lavoie (1995, 1996, 2003) and Dutt (1997). Dumenil and Levy (1999), distinguishing between short and long term, confine the Kaleckian approach to the short run. According to Dumenil and Levy, in the long run, Classical results prevail, that is, the rate of growth and the rate of profits are both positively affected by an increase in savings and negatively affected by an increase in the wage share: supply drives economic growth and profitability through higher saving rates and larger profit shares.

Chick and Caserta (1997) introduce the concept of medium run. According to these authors the long run is irrelevant in the study of real world economies whereas the medium run is a more suitable time framework. Following Chick and Caserta, Kaleckian steady growth analyses apply to the medium run. In our opinion, Chick and Caserta sweep the problem of the long run relevance of Kaleckian G&D models under the carpet. Most of the analyses on growth, elaborated by both mainstream and post Keynesian authors, refer to the enduring tendencies of economies so that the long term seems the most appropriate battleground.

Other Kaleckian authors such as Lavoie and Dutt adopt a different approach. Lavoie (1995, 1996, 2003) attempts to preserve the long run properties of the standard Kaleckian G&D model by making endogenous the normal degree of capacity utilisation and the exogenous component of the investment function. His analysis redefines steady growth equilibrium introducing path dependency

[2] Another objection raised by Marxian and neo Ricardian authors relates to the arguments of the Kaleckian investment function. Bhaduri and Marglin (1990) argue that to capture the role of expected profitability in investment decisions the share of profits in national income is a more suitable variable than the rate of profits, commonly used by Kaleckians. For the same reason, Vianello (1996) suggests to replace the current with the normal rate of profits in the investment function. We do not pursue further this debate. The interested reader could refer to Lavoie (1995).

in the degree of capital utilisation and in the rate of capital accumulation. Lavoie shows that the Kaleckian model so amended reproduces the long run properties of the canonical model. However, the role to entrepreneurs' animal spirits is severely reduced given that producers' accumulation choices are determined within the system from a given set of initial conditions.[3]

Dutt (1997) suggests that hysteresis, which is a form of path dependency, could salvage Kaleckian long run analyses. In particular, this author envisages an economic system in which heterogeneous firms coexist, each firm being characterised by two different levels of the normal degree of capacity utilisation (high and low). Once a particular level is realised, it could persist in response to small shocks. It follows that for such a hysteretic system the normal and the actual degree of capacity utilisation, the former calculated averaging through firms, do not necessarily coincide. An important innovation of this approach is the introduction, via hysteresis, of some nonlinearity into Kaleckian analyses, which are usually based on linear relationships.

We claim that a discrete time structure and nonlinearities could preserve persistent capital underutilisation and the relevance of Kaleckian models of growth and distribution for the study of the enduring tendencies of the economy. In general, Kaleckian analyses adopt a continuous time framework that, for low dimensional dynamical systems, allows only for a small number of possible long term behaviours. A discrete time framework, instead, would allow for a much larger variety of long run behaviours including strongly irregular dynamics even assuming a one-dimensional system. The steady growth path becomes just one of the many possible paths that an economy could travel. Away from such a path it is possible that ever-changing economic circumstances could make insufficient the constant effort of firms to install normal capacity.

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[3] In a private e-mail, Marc Lavoie acknowledges that the relevance of entrepreneurs' animal spirits is diminished within the model with hysteresis compared with the standard Kaleckian one. However, he argues that the role of animal spirits could be reinstated by introducing exogenous (positive or negative) shocks affecting the process according to which entrepreneurs adjust expectations on the rate of growth of demand.

In this paper, we compare two alternative post Keynesian approaches to the analysis of long run economic growth. In the Classical approach, the rate of growth is endogenously determined by households' saving choices and the normal degree of capacity utilisation is exogenous; in the Kaleckian approach, the rate of capital accumulation is driven by an exogenous component, representing the Keynesian animal spirits, and the normal degree of capacity utilisation is endogenous, adjusting through time to the actual one. For both approaches the steady growth equilibrium is characterised by equal current and normal degree of capacity utilisation.[4]

We adopt a common framework for the two approaches based on the following assumptions. Firstly, we frame the analysis in discrete time. Secondly, we introduce an explicit mechanism that incorporates firms' constant effort to bring together the current and the normal degree of capacity utilisation. Thirdly, we introduce a simple nonlinearity into the pricing equation by assuming that the current mark up level depends on the previous period level. This assumption is similar to the one proposed by Dutt (1984) in a continuous time framework according to which the change in the mark up depends on the level of the realised mark up in a point in time.

The scheme of the paper is the following. In section 2 we introduce the basic framework representing an aggregate macroeconomic system; in section 3 we describe the economy in the short run. As shown by Dumenil and Levy (1999), all the Kaleckian results hold in such a context; in section 4 we consider the long run equilibrium for the Classical and the Kaleckian closures of the basic model. In section 5, we develop dynamic analyses. For both the Classical and Kaleckian closures, we show that the steady growth properties do not always hold. Economic growth and profitability could be both demand or supply led depending on the value of the parameters. Section 6 concludes.

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[4] In the paper, we reproduce the standard comparative static results for the steady state of the Classical and Kaleckian closures presented by Amedeo (1986a, 1986b), Committeri (1986), Lavoie (1995, 1996) and Dumenil and Levy (1999).

2. The general framework

The analysis is framed in discrete time: the subscript t refers to the current period, the subscript $t-1$ to the one preceding the current period and the subscript $t+1$ to the one following the current period. We study a single-good closed economy. Technical progress and government intervention are excluded. Production involves a Leontief technology and two factors of production, labour and fixed capital. The labour supply is perfectly elastic and capital does not depreciate. It follows:

$$Y_t^P = \frac{K_t}{a_K} \qquad Y_t = \frac{L_t}{a_L} \tag{1}$$

where Y_t^P represents the full capacity output; K_t physical capital; a_K the reciprocal of the capital technical coefficient; Y_t current output; a_L the reciprocal of the labour technical coefficient and L_t is the labour input.

Production does not involve full capacity utilisation.[5] The degree of capacity utilization, $0 \leq u_t < 1$, is defined as the ratio between current output and full capacity output:

$$u_t = \frac{Y_t}{Y_t^P} \tag{2}$$

Firms set the price, p_t, adding a mark up to prime variable costs,[6]

$$p_t = (1+\mu_t)a_L W_t \quad w_t = \frac{W_t}{p_t} \quad W_t = \tilde{W} \tag{3}$$

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[5] That is, we do not consider the case of a supply constrained economy.

[6] The mark up procedure in equation (3) is widely used in Kaleckian models of growth and distribution. Alternative pricing procedures are discussed in Lavoie (1992, 1995).

where μ_t is the mark up, W_t is the nominal wage rate, which is invariant through time, and w_t the real wage rate.

The above procedure determines income distribution:

$$\pi_t \equiv \frac{r_t K_t}{Y_t} = \frac{\mu_t}{1+\mu_t} \quad 1-\pi_t \equiv \frac{w_t L_t}{Y_t} = \frac{1}{1+\mu_t} \tag{4}$$

where π_t is the profit share and r_t the rate of profits.

We assume that the mark up *level* (and therefore the income shares) at period t depends on the mark up level at period $t-1$ according to the following rule:

$$\mu_t = \Phi(\mu_{t-1}) \tag{5}$$

where $\dfrac{\partial \Phi(\bullet)}{\partial \mu_{t-1}} \geq 0 \quad for \quad \mu_{t-1} \leq \tilde{\mu}$, $\dfrac{\partial \Phi(\bullet)}{\partial \mu_{t-1}} < 0 \quad for \quad \mu_{t-1} > \tilde{\mu}$ and $\Phi(0) = 0$.

The above assumption is similar to the one put forward by Dutt (1984, p. 33) in a continuous time framework according to which the change in mark up is a (non monotonic) function of the level of mark up at a point in time.[7] The economic rationale is that for low levels of the mark up an increase in the current mark up, involving larger market power, allows firms to apply a higher mark up in the next period. However, for mark up levels sufficiently high, a higher current mark up, may correspond to a reduction in market power (because of 'greater entry and falls in concentration ratios') and therefore a lower mark up in the next period.

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[7] In Dutt's analysis the mark up change is also an inverse function of the rate of growth. To keep the analysis simple, we assume the rate of growth does not affect the pricing rule. The main conclusions of the paper are not substantially modified by this assumption.

Firms' investment decisions depend on an accelerator mechanism:

$$\frac{I_t}{K_t} = z_t + \rho(u_t - u_t^n) \tag{6}$$

where $I_t = K_{t+1} - K_t$. According to equation (6), productive capacity is adjusted to the normal degree of capacity utilisation, u_t^n,[8] at a speed ρ. The component z_t of capital accumulation does not depend on those adjustments. Following the Classical interpretation, z_t measures the availability of financial capital necessary to increase productive capacity. Kaleckians, instead, often interpret this component as an expression of entrepreneurs' animal spirits depending on long run (exogenous) expectations (see for example Amadeo, 1986b; and Lavoie, 1996).

From equations (1) and (2), the rates of growth of actual and potential output correspond to

$$g_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}} = \frac{u_t K_t - u_{t-1} K_{t-1}}{u_{t-1} K_{t-1}} \qquad g_t^P = \frac{Y_t^P - Y_{t-1}^P}{Y_{t-1}^P} = \frac{K_t - K_{t-1}}{K_{t-1}} = \frac{I_{t-1}}{K_{t-1}} \tag{7}$$

where the actual rate of output growth, g_t, is smaller, greater or equal to the potential output growth, g_t^P, depending on the changes in the degree of capacity utilisation.[9]

Finally, we introduce a Kaldorian saving function:

$$S_t = s_p r_t K_t + s_w w_t L_t \tag{8}$$

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[8] In the literature the interpretation of u_t^n is not univocal. It could have a technological, a strategic or even a conventional nature. In what follows, for the sake of exposition, we will define it in two radically different ways. First, u_t^n is univocally determined by the existing technology. Second, the normal degree of capacity utilisation is endogenous and it is adaptively adjusted to the current degree of capital utilisation.

[9] Note that in a short or lung run equilibrium capital accumulation, the two rates are necessarily equal.

According to this expression households save out of profits and wages according to fixed proportions, where s_p and s_w represent the propensity to save out of profits and the propensity to save out of wages respectively and $0 \leq s_w < s_p \leq 1$.

3. The economy in the short run

In the short run, productive capacity and long run expectations are given. It follows that $K = \overline{K}$, $u_t^n = \overline{u}^n$ and $z_t = \overline{z}$. Moreover, adjustments between supply and demand (saving and investment) occur through changes in production levels:

$$Y_t - Y_{t-1} = \theta(I_{t-1} - S_{t-1}) \tag{9}$$

Firms' short run investment demand corresponds to:

$$I_t = (\gamma + \rho u_t)\overline{K} \tag{10}$$

where $\gamma = \overline{z} - \rho\overline{u}^n$.

We assume, moreover, that firms do not change the mark up in the short run, that is,

$$\mu_t = \overline{\mu} \tag{11}$$

In a short run equilibrium supply equals demand:

$$S_t = I_t \tag{12}$$

If condition (12) holds, we can derive the short run equilibrium solutions:

$$\pi^{SR} = \frac{\bar{\mu}}{1+\bar{\mu}} \quad u^{SR} = \frac{a_K \gamma}{(s_p - s_w)\pi^{SR} + s_w - a_K \rho} \quad g^{SR} = \frac{I}{\bar{K}}^{SR} = \left(\gamma + \rho u^{SR}\right) \quad r^{SR} = \frac{\pi^{SR} u^{SR}}{a_K} \tag{13}$$

For short run equilibria the properties of the standard Kaleckian models hold (see Lavoie, 1996; and Dumenil and Levy, 1999). In particular, according to the paradox of thrift, the rate of profits and the rate of growth decrease with an increase in average savings:

$$\frac{dr^{SR}}{ds_P} = \frac{\pi^{SR}}{a_K} \frac{du^{SR}}{ds_P} < 0$$

$$\frac{dg^{SR}}{ds_P} = \rho \frac{du^{SR}}{ds_P} < 0$$

where $\frac{du^{SR}}{ds_p} = -\frac{a_K \gamma \pi^{SR}}{\left[(s_p - s_w)\pi^{SR} + s_w - a_K \rho\right]^2} < 0$.

According to the paradox of costs, the rate of profits and the rate of growth decrease (increase) with an increase in the profit share (wage share):

$$\frac{dr^{SR}}{d\bar{\mu}} = \frac{s_w - a_K \rho}{\left[(s_p - s_w)\pi^{SR} + s_w - a_K \rho\right]^2} < (\geq) 0 \quad for \quad s_w < (\geq) a_K \rho$$

$$\frac{dg^{SR}}{d\bar{\mu}} = \rho \frac{du^{SR}}{d\bar{\mu}} < 0$$

where $\frac{du^{SR}}{d\bar{\mu}} = -\frac{a_K \gamma (s_p - s_w)}{\left[(s_p - s_w)\pi^{SR} + s_w - a_K \rho\right]^2 (1+\bar{\mu})^2} < 0$

The short run equilibrium (13) is stable if and only if:[10]

$$-2 < \theta\left\{a_K \rho - \left[(s_p - s_w)\pi^{SR} + s_w\right]\right\} < 0 \tag{14}$$

4. The economy in the long run

The long run is represented as a sequence of short run equilibria. We propose and compare two alternative closures for the model presented in Section 2. In the Classical closure, firms' decisions concerning capital accumulation are linked to the financial funds made available by households' saving decisions. Moreover, firms adjust the current to the normal level degree of capacity utilisation, the latter being univocally determined by the existing techniques of production. In the Kaleckian case, instead, capital accumulation is governed by long run expectations concerning the rate of growth of demand. The normal degree of capacity utilisation, which is determined by conventional or strategic factors, adjusts through time to the current degree of capacity utilisation.

4.1. The Classical closure

Firms investment plans are constrained by the quantity of financial funds made available by households' savings:

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[10] The short run dynamics corresponds to the following one-dimensional linear map:

$$u_t = \Lambda(u_{t-1}) = u_{t-1}\left\{1 + \theta\left[\rho a_K - (s_p - s_w)\pi^{SR} - s_w\right]\right\} + \theta\gamma a_K$$

$\Lambda(u_{t-1})$ is stable as long as $-1 < \Lambda'(u^{SR}) < 1$. These inequalities correspond to condition (14).

$$z_t = z_{t-1} + \phi(\sigma_{t-1} - z_{t-1}) \tag{15}$$

where σ_t is the rate of growth of savings, that is, the savings/capital ratio and ϕ the speed at which the banking sector provides credit to firms.

The normal degree of capacity utilisation is determined by the technology and it is invariant over time:[11]

$$u_t^n = \tilde{u}^n \tag{16}$$

The long run is fully described by equations (1)-(8), (15) and (16). There are two long run equilibria corresponding to the cases $\mu = 0$ and $\mu > 0$:

$$u = \tilde{u}_n \quad \mu = \Phi(\mu) \quad \pi = \frac{\mu}{1+\mu} \quad \frac{I}{K} = g = \left[(s_p - s_w)\pi + s_w\right]\frac{\tilde{u}_n}{a_K} \quad r = \frac{\pi\tilde{u}^n}{a_K} \tag{17}$$

Discarding the case $\mu = 0$,[12] we now move on comparing long run steady growth paths:

$$\frac{dg}{ds_p} = \frac{\pi\tilde{u}^n}{a_K} > 0$$

[11] To keep the analysis simple, we assume that only a technique of production is available to firms. This technique involves an optimal degree of capacity utilisation corresponding to $\tilde{u}^n$. Typically, the normal degree of capacity utilisation does not correspond to full capacity utilisation, that is, $\tilde{u}^n \neq 1$ (for example due to the turn over of machines for maintenance). Allowing for a larger set of techniques would make the normal degree of capacity utilisation endogenous depending on distribution. The choice of a degree of plant utilisation as part of the overall problem of choice of techniques has been explored by Kurz (1986).

[12] The case $\mu = 0$ corresponds to

$$\pi = 0 \quad \frac{I}{K} = g = \frac{s_w\tilde{u}_n}{a_K} \quad r = 0$$

This solution resembles a 'perfectly competitive' in which firms do not have market power.

That is, accumulation is driven by households' thriftiness: the paradox of thrift is overturned.

In order to verify the paradox of costs, we consider a positive shift in the pricing function. Denoting α the shift parameter, we have that

$$\frac{dg}{d\alpha}=\frac{\tilde{u}^n}{a_K}(s_p-s_w)\frac{1}{(1+\mu)^2}\frac{d\Phi(\mu)}{d\alpha}>0 \quad \frac{dr}{d\alpha}=\frac{\tilde{u}^n}{a_K}\frac{1}{(1+\mu)^2}\frac{d\Phi(\mu)}{d\alpha}>0$$

That is, also the paradox of costs does not hold along a steady growth path (see Lavoie, 1996; and Dumenil and Levy, 1999).

4.2 The Kaleckian closure

In the Kaleckian closure, z_t represents the expected rate of growth of demand. Assuming that expectations are driven by entrepreneurs' (exogenous) animal spirits, we can write

$$z_t=\tilde{z} \tag{18}$$

Moreover, firms revise their conception of the normal degree of capacity utilization as follows

$$u_t^n=u_{t-1}^n+\lambda(u_{t-1}-u_{t-1}^n) \tag{19}$$

The long run is fully described by equations (1)-(8), (18) and (19). As in the previous model there are two long run equilibria, corresponding to the cases $\mu=0$ and $\mu>0$:

$$\mu=\Phi(\mu) \quad \frac{I}{K}=g=\tilde{z} \quad \pi=\frac{\mu}{1+\mu} \quad u=\frac{a_K\tilde{z}}{(s_p-s_w)\pi+s_w} \quad r=\frac{\pi\tilde{z}}{(s_p-s_w)\pi+s_w}$$

Discarding the solution equal to zero,[13] the comparison of long run equilibria gives

$$\frac{du}{ds_p} = \frac{-a_K \tilde{z}\pi}{[(s_p - s_w)\pi + s_w]^2} < 0 \qquad \frac{dr}{ds_p} = \frac{\pi}{a_K}\frac{du}{ds_p} < 0$$

$$\frac{du}{d\alpha} = \frac{(s_p - s_w)a_K \tilde{z}}{[(s_p - s_w)\pi + s_w]^2 (1+\mu)^2}\frac{d\Phi(\mu)}{d\alpha} < 0$$

$$\frac{dr}{d\alpha} = \frac{s_w \tilde{z}}{[(s_p - s_w)\pi + s_w]^2 (1+\mu)^2}\frac{d\Phi(\mu)}{d\alpha} < 0 \quad for \quad s_w \geq 0$$

That is, considering the impact of changes in s_p and α on the rate of profits, the paradox of thrift holds whereas the paradox of costs does not. Moreover, as long as $\tilde{z}$ is exogenous, the rate of growth is unaffected by changes in these parameters (see Lavoie, 1995).

5. Dynamics

In order to study the long run dynamics, we choose the following explicit form for the pricing equation (5):

$$\Phi(\mu_{t-1}) = \alpha \mu_{t-1} e^{\frac{\beta - \mu}{\beta}} \tag{20}$$

[13] The case $\mu = 0$ corresponds to

$$u = \frac{a_K \tilde{g}}{s_w} \qquad \frac{I}{K} = g = \tilde{z} \qquad \pi = 0 \qquad r = 0$$

For an interpretation of this solution, see the previous note.

where $\tilde{\mu} = \beta$ and $\Phi(\tilde{\mu}) = \alpha\beta$. Equation (20) captures a typical feature of real economies, that is, stickiness in price formation: the slope of the curve is higher (smaller) in absolute value for $\mu_{t-1} < (>) \beta$. That is, prices increase faster than they decrease. Equation (20) is represented in Figure 1.

The dynamic properties of both the Classical and the Kaleckian closures depend crucially on equation (20), which possesses all the properties of a unimodal map.[14] Two steady growth equilibria correspond to this map, that is, $\mu = 0$ and $\mu = \beta(1+\ln(\alpha)) > 0$. The local stability condition of both these equilibria is

$$-1 < \Phi'(\mu) = \alpha\left(\frac{\beta-\mu}{\beta}\right)e^{\frac{\beta-\mu}{\beta}} < 1 \tag{21}$$

For $\mu = 0$, condition (21) reduces to $\alpha e < 1$. The positive equilibrium, instead, is stable as long as $\alpha < e$.

The inequalities (21) hold depending exclusively on the shift parameter α. Changes in α have also an impact on the global dynamics properties of the map (20). Figure 2 is a bifurcation diagram showing the long run behaviour of μ_t as α is increased. For, $\alpha e < 1$ only the equilibrium $\mu = 0$ exists, which is stable. When $\alpha = e^{-1}$, this equilibrium loses stability via a Saddle node bifurcation. In correspondence of $\alpha = e^{-1}$ a new equilibrium is created, that is, $\mu = \beta(1+\ln(\alpha)) > 0$. The positive equilibrium is stable for $e^{-1} < \alpha < e$. At $\alpha = e$, it loses stability through a period doubling (or Flip)

[14] A map is the way is usually called in the theory of dynamical systems a difference equation. A flow, instead, corresponds to a differential equation. The reader may refer to Devaney (1989) for an introduction to nonlinear dynamical systems.

bifurcation. Increasing α further gives rise to cycles of any order and eventually to chaotic dynamics.

Figure 3, which has been plotted for $\beta = 0.1$ and starting from the initial condition $\mu_0 = 0.3$, shows how increasing α above e – that is, for those values of the shift parameter giving rise to cyclical or chaotic behaviour for the map (20)– affects the average mark up and, via equations (4), the average shares of profits and wages in income distribution: whereas the average mark up increases throughout with the shift parameter α, the average profit share increases over an initial range of α values and then behaves irregularly, increasing or decreasing for higher values of the shift parameter. The behaviour of the average wage share as α is varied is symmetric to the behaviour of the average profit share increasing (or decreasing) when the latter is decreasing (or increasing).

We now examine separately the Classical and the Kaleckian closures.

5.1 The Classical closure

We assume for our simulations the parameter values: $s_w = 0.175$, $s_p = 0.4$, $a_K = 2.75$, $\tilde{u}^n = 0.7$, $\rho = 0.25$, $\beta = 0.1$, $\phi = 0.5$ and $0 < \alpha < 10$; and the initial conditions $\mu_0 = 0.3$, $z_0 = 0.04$, $u_0 = 0.7$ and $K_0 = 10$. Given the parameter values constellation, it follows that for $\mu = \beta(1+\ln(\alpha))$, we have: $0 < \mu < 0.33$, $0 < \pi < 0.248$, $u = 0.7$, $0.045 < g < 0.059$, $0 < r < 0.0.063$; and for $\mu = 0$, we have: $\pi = 0$, $u = 0.7$, $g = 0.045$, $r = 0$. Figure 4 shows how the average values of u_t, r_t and g_t change as α is varied between e and 10. As Figure 4 shows, there are intervals of α for which the average rate of profits and the average rate of growth decrease as the shift parameter is increased. In these intervals, the economy does not behave as predicted by the comparison of steady growth equilibria. This is due to the fact that shifts in the pricing equation do not necessarily involve, as in steady growth equilibrium, increases in the profit share or, equivalently, decreases in the wage share. Those shifts may destabilise the dynamic behaviour of the economic variables we are referring to and give rise to fluctuations the average values of which differ from the corresponding steady growth values. What emerges from the comparison between Figures 3 and 4 is that, considering averages, the behaviour of r_t follows closely the behaviour of π_t; whereas, as far as g_t is concerned, there are ranges of values of α for which economic growth is wage driven. The latter result contradicts the steady growth properties of the Classical closure. [15]

Note finally that, the average degree of capacity utilization is only subject to small fluctuations gravitating around the normal value.

5.2 The Kaleckian closure

The parameter values we choose for this case are: $s_w = 0.15$, $s_p = 0.4$, $a_K = 2.75$, $\tilde{z} = 0.05$, $\rho = 0.01$, $\beta = 0.1$, $\lambda = 1$ and $0 < \alpha < 10$;[16] and the initial conditions are $\mu_0 = 0.3$, $u_0^n = u_0 = 0.7$ and $K_0 = 10$. From the parameter values constellation it follows that for $\mu = \beta(1 + \ln(\alpha))$ we have: $0 < \mu < 0.33$, $0 < \pi < 0.248$, $0.596 < u < 0.786$, $g = 0.05$, $0 < r < 0.054$; and for $\mu = 0$, we have $\pi = 0$, $u = 0.786$, $g = 0.05$, $r = 0$. Figure 5 plots the average values of u_t, r_t and g_t for the interval $e < \alpha < 10$.

As Figure 5 shows, there are intervals of α for which the average rate of profits and the average rate of growth decreases as the shift parameter is increased. In these intervals, the economy does not behave as predicted by the comparison of steady growth equilibria. Moreover, for values of α

[15] Simulations, not presented here, show that higher values of s_p and s_w determine higher average rates of profits and growth. That is, the paradox of thrift always holds as in the steady growth equilibrium.

[16] Compared to analysis presented in the previous Section, the speed of adjustment ρ is much smaller, this is to avoid the upper bound constraint on the degree of capacity utilisation being binding.

sufficiently close to e the average rate of profits moves in the opposite direction of the average profit share; and for values of α sufficiently close to 10 the average rate of output growth also moves in the opposite direction of the average profit share. That is, for values of the shift parameter sufficiently high economic growth is wage led. These results do not always allow to confirm the steady growth properties of the Kaleckian closure.[17]

6. Conclusions

In this paper we explored two alternative post Keynesian approaches to the analysis of long run economic growth. To facilitate the comparison we used a common framework based on the following assumptions: Firstly, we framed the analysis in discrete time. Secondly, we introduced an explicit mechanism incorporating firms' constant effort to bring together the current and the normal degree of capacity utilisation to take into account the neo Ricardian critique to the Kaleckian approach. Finally, we introduced a simple nonlinearity in the pricing equation.

In the Classical closure accumulation is governed by households' saving choice and the normal degree of capacity utilisation is exogenous; in the Kaleckian closure capital accumulation is driven by entrepreneurs' (exogenous) animal spirits and the normal degree of capacity utilisation is endogenous. The latter adjusts through time to the actual one on the basis of entrepreneurs' expectations. In both cases the steady growth equilibrium is characterised by normal capacity utilisation.

[17] Simulations, not presented here, show that higher values of s_p and s_w determine smaller average rates of profits. Moreover, a higher value of s_p determine a higher average rate of growth whereas a higher value of s_w determine a lower average gt. That is, not always the paradox of saving holds for the Kaleckian closure.

We also explored the dynamic properties of the model. For both the Classical and the Kaleckian closures, we have shown that, when the steady growth path is not an attractor, the comparative statics properties of the model are not satisfied. That is, once we allow for nonlinearities and for a discrete time framework, the mechanisms governing economic growth and profitability crucially depend on the value of the parameters.

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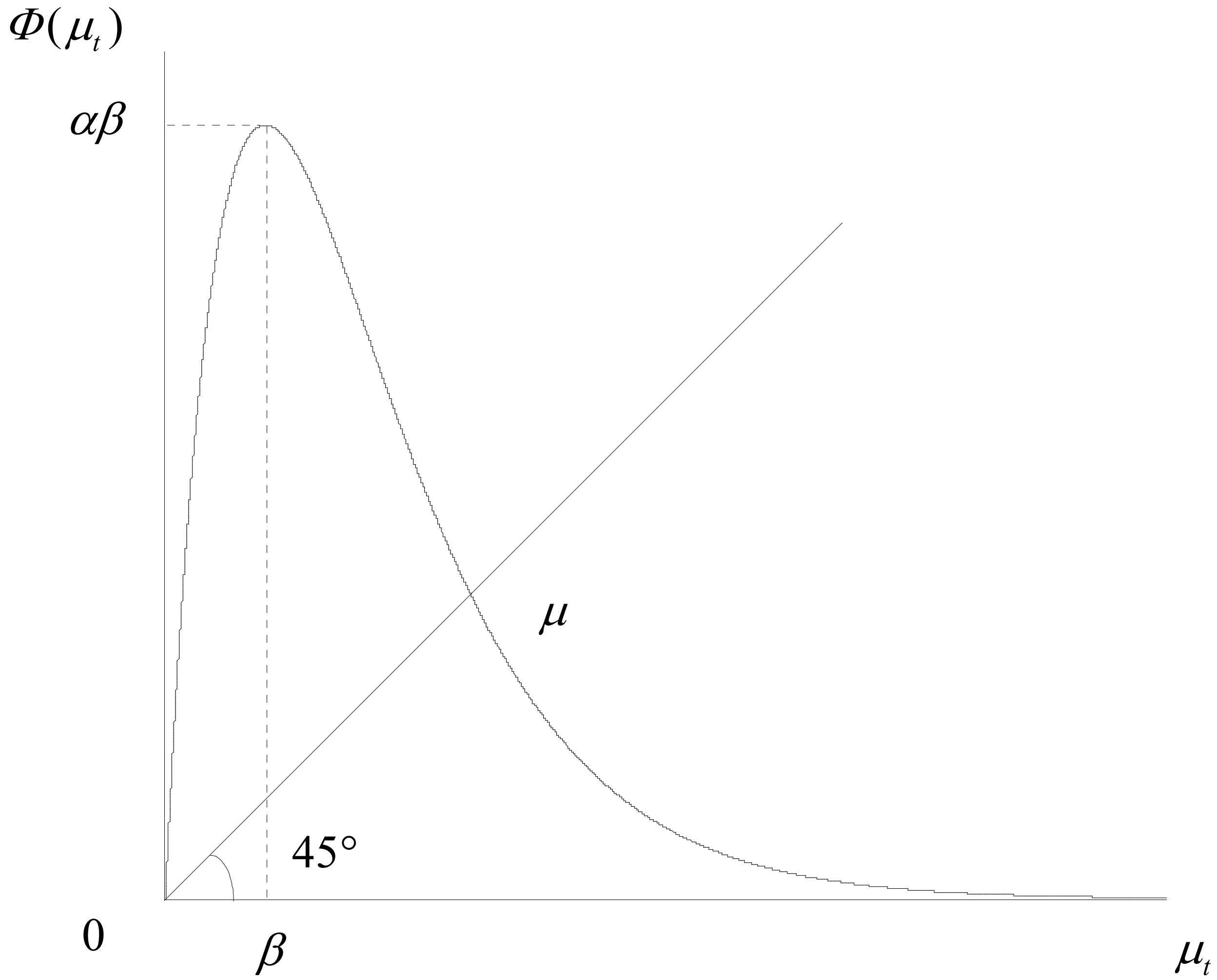

Figure 1

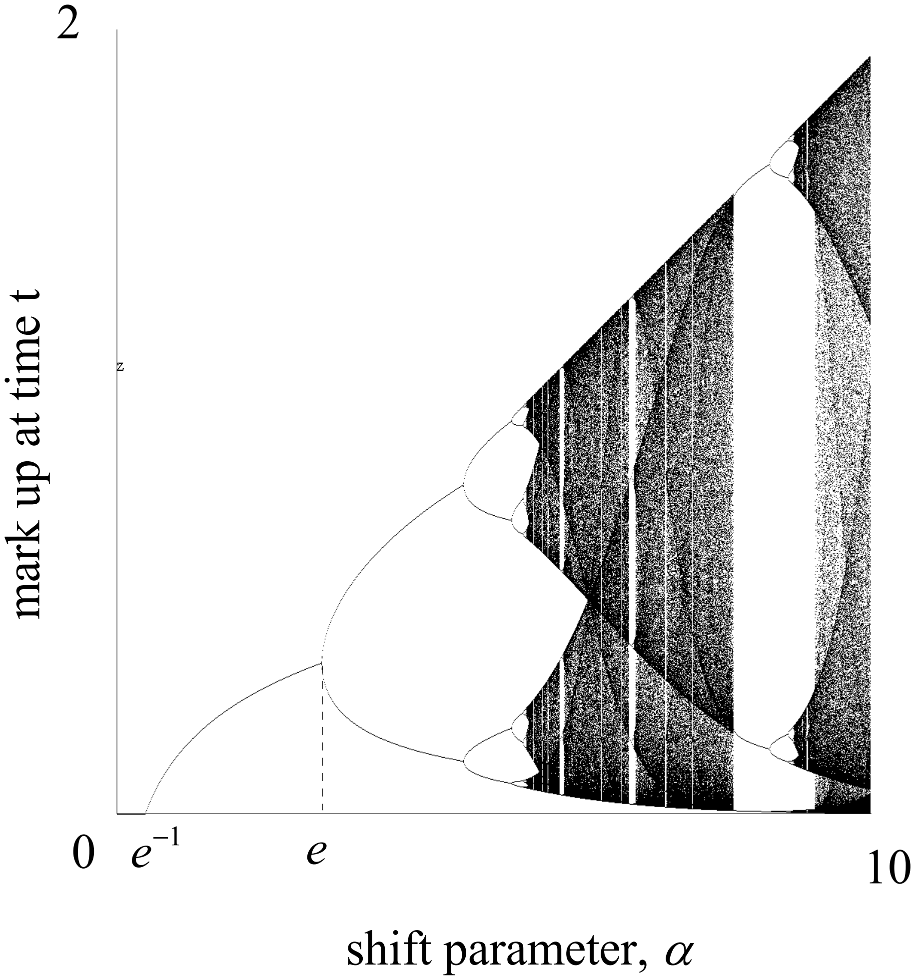

Figure 2

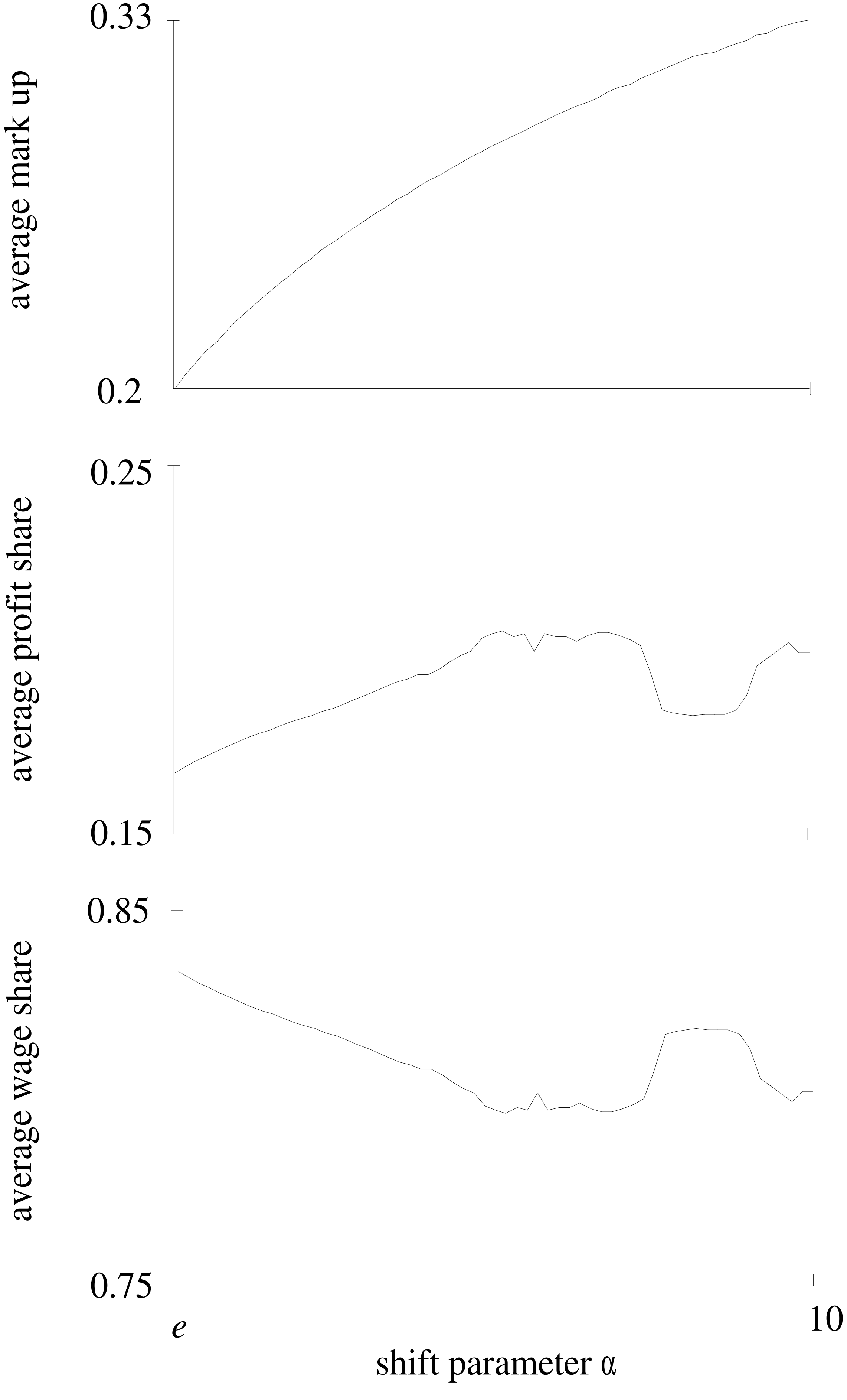
0.33
average mark up
0.2
0.25
average profit share
0.15
0.85
average wage share
0.75
e
10
shift parameter α

Figure 3

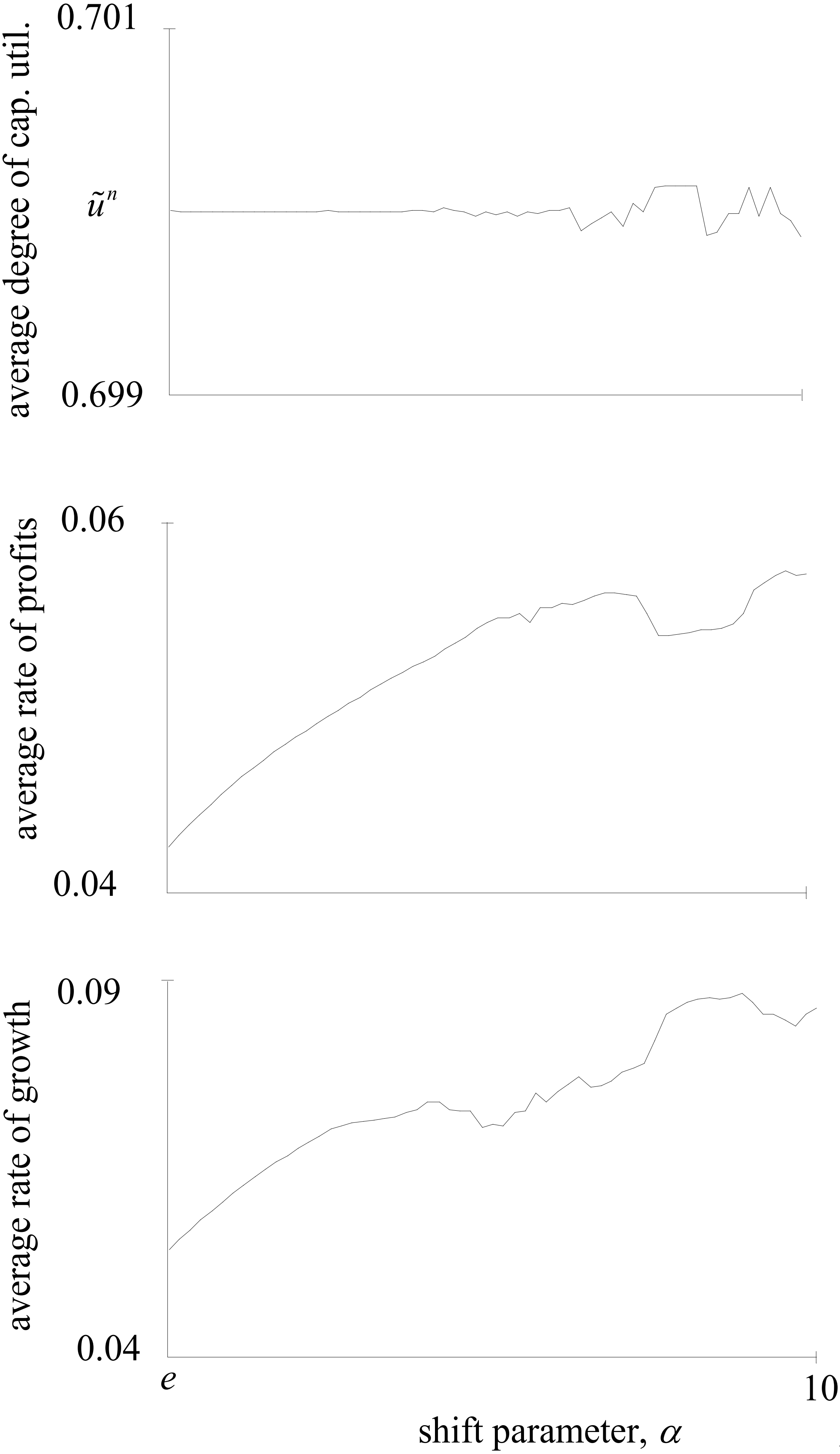
0.701
average degree of cap. util.
$\tilde{u}^n$
0.699
0.06
average rate of profits
0.04
0.09
average rate of growth
0.04
e
10
shift parameter, α

Figure 4

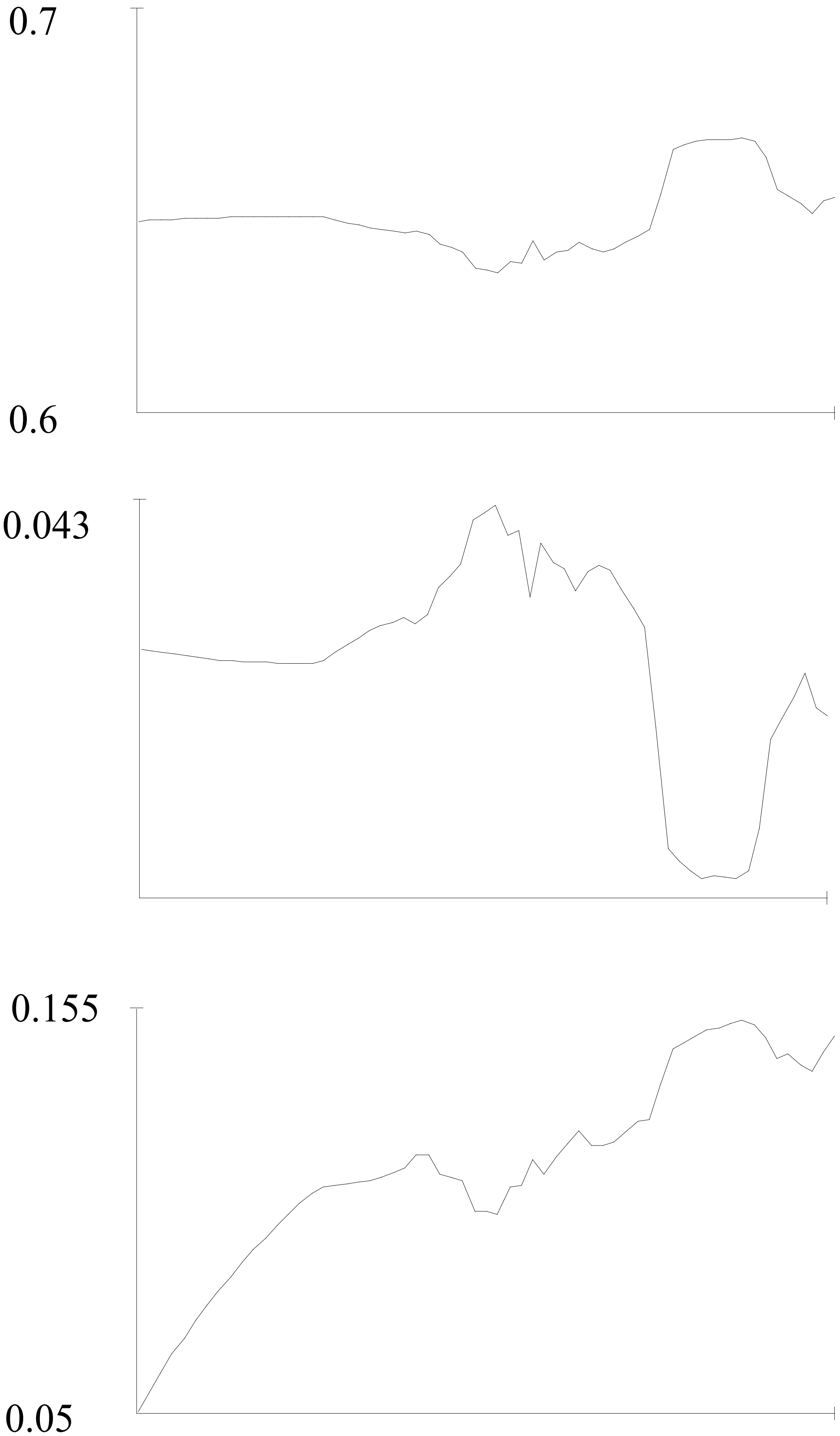

Figure 5