Endogenous Lifespan, Health Funding and Economic Growth

Davide Dottori[a]

Abstract

An over lapping generations model is set up to compare in terms of economic growth rates and income distribution two regimes of health funding: private and public ones. Health is not only a component of human capital but it also yields directly utility and - by enlarging lifespan - it reduces future discounting thus affecting the propensity to invest in human capital accumulation. In the private system health expenditure is chosen in a decentralized way, whereas in the public regime it is provided by government and funded through an income tax, with agents voting over the tax rate. Endogenous poverty and low development traps are shown to may arise. Inequality turns out to decline faster under public regime, whereas in the private one it may be non-decreasing. Private system generally results to bring about higher growth rates, but when income distribution is enough uneven public system may feature higher growth rates.

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[a]Dipartimento di Economia, Universit Politecnica delle Marche, E-mail: d.dottori@univpm.it .

1 Introduction

1.1 Health and Economic Growth

Still not much has been said about effects of health on economic growth, though health is widely recognized to be an unavoidable part of human capital and a prerequisite for sustained economic growth.

The aspect that has mainly been stressed in literature is the positive effect of health on productivity growth: Bloom *et al.*[4] found that good health has a positive and statistically significant effect on labor outcome. Fogel [11] estimates that nutritional improvements alone contributed about 20 -30 % of income growth per capita not explained by factors accumulation in Britain during 1780-1979. Mayer Foulkes [18] shows the positive effect of early child health and on the probability of obtaining a higher education later in life. In a recently released report, the Micronutrient Initiative and UNICEF [20] find that the impacts of food deficiencies on mortality, sicknesses, and on early child, mother and adult cognitive ability are significant. Adult health contributes directly to adult income: healthier workers may be productive, or at least less likely absent because of illness (or illness of their family's member). Howitt [16] highlights the beneficial effects of helath on creativity, learning capacity and copying skills.

Of course, there is also a relation from income to health condition, since richer countries can afford higher health services. Nevertheless Devlin and Hansen [10] find Granger causality running in both directions between health and GDP in OECD countries. Income is an important determinant of health expenditure first of all because people demand the "good health" since it brings them direct utility, even if the concept of health care is not narrowed to basic needs. Therefore health should be included as an argument of utility function since it is important in itself for agents: when they demand for health goods or facilities actually they demand for health well-being. In this respect Van Zon and Muysken[21] suggest that health can even become a "substitute for growth" as far as resources potentially available for other kind of consumption goods can be diverted toward health sector. Asa and Pueyo[2] consider the trade off between life expectancy effect and this kind of competition for resources finding that a crucial role is played by longevity response to an increase in health expenditure. Not least, general health condition is a relevant aspect of the degree of development of a country, indeed they have been included in the Human Development Index (Human Development Report[19]).

Anyway, health is likely to have also another not negligible indirect role which can affect economic growth. Unlikely any other element of human capital, it increases lifespan thus lowering individuals discount rate, that is they are effectively more patient and willing to invest, thus boosting economic growth. The expansion of life expectancy allows for returns to be obtained over a longer period of time, thus encouraging accumulation of human capital (de la Croix and Licandro[8]), while high mortality rates reduce returns in education, where risk is less diversifiable (Chakraborty[5]).

Savings and investment rates are generally low in high mortality societies because they are more risky and because the returns from them are expected to be received along a shorter horizon. Fuchs [12] finds empirical evidence in support of the positive link between individual patience rates and health. Moreover, if economies with high childhood mortality are considered, the possibility itself to achieve adulthood or old age can be considerably influenced by health condition, so in less developed countries, a reduction in mortality rates tends also to increase the labour force. In this respect the situation is different for most developed countries where as Asa and Pueyo[2] point out improvements in life expectancy makes the retirement period longer without increasing labour force. de la Croix and Licandro[8] suggest that this aging effect make also introduce some obsolescence in human capital if the economy consists of more old agents who did their schooling a long time ago (de la Croix and Licandro[8]).

In Fig. 1 data from World Health Report [23] have been used to plot the increasing relation between health expenditure and life expectancy at birth. Chakraborty and Das [6], Chakraborty[5], Leung and Wang [17] model the discount factor as a function of health investment, that exhibits properties consistent with such a plot.

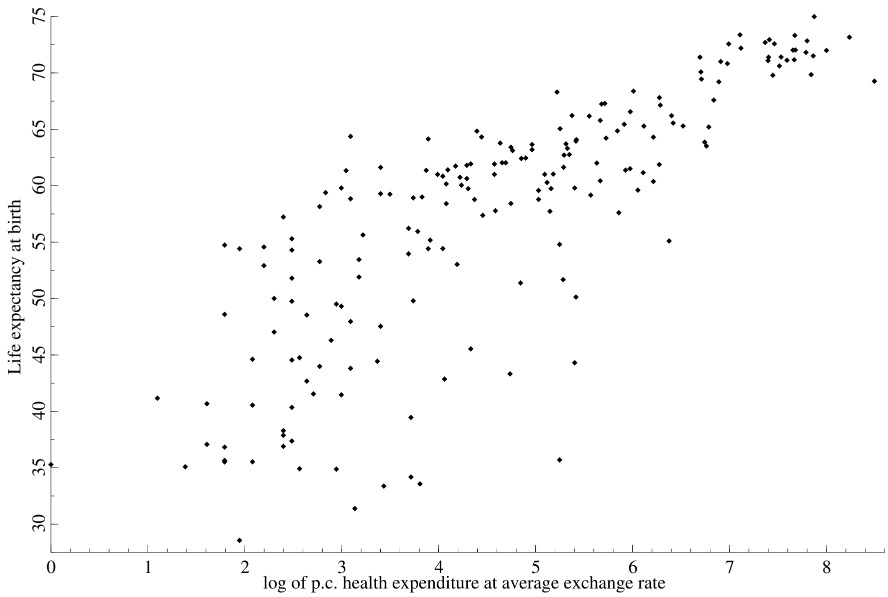


Figure 1: Data source: World Health Report, 2004

As far as relations between health and inequalities are concerned, there is again a a feed back relation between health and income inequalities: the causal direction from the inequalities in income to inequalities in health has been mostly studied (e.g. see Deaton [9] for a critical survey), but less has been said about the opposite direction: Galor and Mayer [13] show how health inequality may be a factor that explains persistence and increasing income inequalities, if health and education are complementary goods, but a minimum level of health is necessary and borrowing constraints are present. Chakraborty and Das [6] put in evidence that in absence of perfect annuities markets[1] health shocks can have persistent effects on income distribution; Chakraborty[5] shows how “development traps” and persistent inequality may arise when mortality risk is endogenous and negatively dependent on health investment.

Finally health exhibits also some features of a public good since it involves externalities and non-rivalries and non-excludibility aspects: for example policies through investment in sanitation, vaccination and nourishment, may reduce the impact and the incidence of diseases, and the population size at risk of contacting these diseases. In this sense health improvements can exhibit spillover effects, especially where epidemics and diseases are a concrete obstacle to economic activity (Barro[3]).

1.2 Private *vs* Public Health Funding

In light of all these features of health, it might be interesting to investigate the effects on inequalities and on economic growth of two different regimes of health funding: public and private. Hosoya [15] focuses on public health system studying the problem of optimal tax rate, however in his model health is an exogenous variable at individual level, and there is not a real comparison between the two systems. A comparison between a public and a private regime is conducted by Glomm and Ravikumar[14], but it is applied to educational capital in a model in which formal schooling is the engine of growth.[2] Since health is a component of human capital their model is a natural reference for this analysis. What they mainly conclude is: *(i)* private regime yields higher growth; only if income inequalities is high enough then public system may perform higher growth for short future periods; *(ii)* income inequality declines faster under public education than under private education. Actually, point *(i)* relies on the time-allocation solution they find: in private case, each agent accounts that an additional unit of time spent toward education increases not only his earnings but also the bequests passed on to his offspring, while in the public case, the latter benefit is not taken into account, hence investment in human capital is systematically higher in private regime.

Now, if health shares with education several features since both are important components of human capital, it owns also some peculiarities, such as the effect on discount factor and the relevance in utility[3]. Hence, keeping the same structure of their model, some of these peculiarities are introduced in order to investigate if and how results change. In this model health plays mainly three roles: it is an argument of utility function, increases directly the productivity and affects life expectancy. This last feature

[1]Perfect annuities markets mean that all savings are intermediated through mutual founds that invest them in assets. The gross returns from these investments are then distributed among the surviving old.

[2]de la Croix and Doepke[7] extend the analysis to a framework where also fertility decisions are endogenous.

[3]Also education can provide utility directly (“pleasure of studying”), but reasonably this direct effect is much more relevant about health.

affects also the discount factor. Since time invested on education increases if agents discount less the future, the decision about how much studying in private system is no longer the same across individuals as in Glomm and Ravikumar, but it depends on the health investment they can afford. Agents with low income may be induced from public provision of health to accumulate more human capital they would do under private system. On the other side, in private system more implications are taken into account, while in public regime health spending turns out to be proportional to average income which is considered as exogenous by an agent when she votes for her preferred tax rate.

As far as inequalities are concerned, public system turns out to reduce them more quickly than private system. Actually in private system inequalities are not reduced at all if long run returns are not decreasing. Moreover the model has an endogenous source of poverty trap, that relies on the growth enhancing - but at a decreasing rate - effect of health on life expectancy: at low initial conditions health improvements have a big impact on survival probability and hence on the incentive to invest in human capital accumulation; then further improvements in health has a smaller and smaller impact on life expectancy. This can cause a temporary convexity in human capital map function, an unstable equilibrium and a poverty trap mechanism to take place.

The rest of the paper is organized as follows: in Section 2 the basic framework for the model is illustrated; in Section 3 and Section 4 optimizing choices and dynamics from private and public system respectively are obtained. Section 5 deals with the comparison between the two regimes when agents are homogeneous, while in Section 6 the heterogeneous case is concerned. Section 7 provides some simulated examples after that parameters have been calibrated. Finally in Section 8 there are some conclusive remarks.

2 The Basic Framework

The model consists in an overlapping generations economy where agents live for two periods. The probability of living during the second part of life depends positively on health. In the first period agents allocate their fixed amount of time between leisure and learning activities. At the end of first period they have a child. Agents live only a fraction ϕ of second period. During this interval they work, and allocate their income between consumption good and health care facilities.

Each agent is endowed with a stock of human capital, that depends on the parents' stock of human capital, on time devoted to learning and on her health during her working period[4]. Agents are heterogeneous with respect to their endowments in human capital,

[4]In Section 1 has been highlighted that also health before the productive age have an influence on labour productivity. Anyway in this framework this would add one dimension to optimization problem making less tractable the model without adding any decisive insight.

whose distribution is initially given. Inequalities are measured by the variance of human capital distribution, that in this model coincides with the variance of income distribution.

In this model, generations are linked through two channel: on one side every agent's stock of human capital depends on the stock of her parents; on the other side, health expenditure of the household has a positive effect on life expectancy of the children. This introduces an externality since the parents do not internalize the benefit that their health expenditure has on the discount factor of the children through their life expectancy. Differently from GR there is no specific bequest motive in utility function[5]. In the private system each agent decides in autonomy her preferred health expenditure; instead under public regime health is provided by government and is the same for all individuals. This health expenditure is financed by levying a proportional tax on labour income, with the tax rate determined through majority voting.

In order to keep things easy to handle, very simple functional forms are assumed: logarithmic utility function, Cobb Douglas production function for human capital, and final good production linear in human capital so that the wage rate can be normalized to one. Agents receive utility from leisure and from a mix of health and consumption. Anyway consumption and health in the first period are assumed to be variables out of the agent's control since they are decided by parents: they can be thought to be included in their parents's consumption and health. Hence, objective function can be formalized as:

$$U = \ln n_t + \phi(h_t)[\ln c_{t+1} + \gamma \ln h_{t+1}] \tag{1}$$

where n_t, c_{t+1} and h_{t+1} are the control variables and denote respectively leisure, consumption and health. The parameter γ represents the weight of health relative to consumption: the greater is γ the greater is the importance of health relative to consumption.

The survival probability, $\phi(h_t)$, is a function of health investment during the first part of her life, i.e. the "health care environment" in her family. This health investment can be thought to occur through net food intake, personal care and hygiene, accessing clinical facilities and related medical expenditure that is the key to mortality reduction[6]. Subjective discounting is ignored to keep the notation as simple as possible, although it can be easily incorporated. Learning is assumed to have a time cost in terms of less leisure, for the sake of simplicity we abstract from monetary costs.

Looking at the evidence from Fig.1 , and following Chakraborty and Das [6], the function $\phi(\cdot)$ should exhibit the following properties:

$$\phi(\cdot) \in (0;1), \quad \phi' \geq 0, \quad \phi'' \leq 0, \qquad \text{and} \quad \lim_{h\to\infty} \phi(h) = \overline{\phi} \leq 1.$$

[5] Aguiar-Conraria [1] shows how the conclusion that private system leads to higher growth is not necessarily true if altruism is not allowed.

[6] Actually also on an "innate" component given by nature should play a role: $\phi^i = \phi(h^i, \kappa^i)$ where κ^i is the innate health capital of agent i. Anyway it is assumed for the sake of simplicity that $\kappa^i = 1$ for all i, so that $\phi(h^i) \equiv \phi(h^i, 1)$.

Health investment augments lifespan, but at a decreasing rate, so that even when health expenditure is very high the limit for $\phi(\cdot)$ is finite and no greater than one since it is a probability. Some explicit forms for $\phi(h_t)$ are available in literature. Chakraborty and Das use:

$$\phi(h) = \begin{cases} ah^{\epsilon} & \text{if } h \in [0, \hat{h}] \\ \bar{\phi} & \text{otherwise} \end{cases} \tag{2}$$

where $\hat{h} \equiv (\bar{\phi}/a)^{1/\epsilon}$, $\epsilon \in (0; 1)$ and a is a positive parameter related to exogenous medical progress[7].

Another example for an explicit $\phi(h)$ is available in Chakraborty [5]:

$$\phi(h) = \beta \frac{h}{1+h} \tag{4.2a}$$

In this case, $\beta = \bar{\phi}$.

Finally Leung and Wang [17] propose

$$\phi(h) = p_0 + \bar{p}\sqrt{\frac{h}{1+h}} \tag{4.2b}$$

where p_0 represents the minimum level of longevity and $\bar{p}$ can be thought as the state of the art of medical progress. In this case the supremum for $\phi(\cdot)$ is $\bar{\phi} = p_0 + \bar{p}$.

About the evolution of human capital, ξ, the following functional form is assumed

$$\xi_{t+1} = \psi(1 - n_t)^{\lambda} h_{t+1}^{\theta} \xi_t^{\nu} \tag{3}$$

where $\psi > 0$ is a productivity parameter; $(1 - n_t)$ denotes time devoted to learning in youth and h_{t+1} represents the current health investment. The idea is that to accumulate human capital some favorable health conditions are necessary together with time devoted to study and the inherited stock of human capital. The inclusion of lagged human stock is quite standard since it represents the intergenerational evolution of knowledge, skills, and so on.

As in GR production technology is linear in human capital so that wage rate can be normalized to one and every individual's income is equal to her human capital. The distribution of human capital is initially given. Dynamics of the variance will be used to measure dynamics of inequalities.

Before studying the private system, it may be convenient to briefly summarize the three channels through which health plays a role in this simple model: it provides directly utility, it increases lifespan and finally it enhances productivity and income.

[7] Actually it is a little bit weird assuming medical progress exogenous with respect to health investment; anyway a_t can be seen as a parameter reflecting all the factors involved in life expectancy not directly related to health investment.

3 Private System

In the private system, the optimization problem consists in maximizing (1) with respect to n_t, c_{t+1} and h_{t+1} under (3) and the budget constraint[8]:

$$\xi_{t+1} = c_{t+1} + h_{t+1} \tag{4}$$

with h_t and ξ_t given.

Using (4) to replace c_{t+1} in (1) and using (3) to substitute ξ_{t+1}, the constrained optimization problem is equivalent to maximize with respect to n_t and h_{t+1} the following objective function:

$$\ln n_t + \phi(h_t)\{\ln[(\psi(1-n_t)^{\lambda} h_{t+1}^{\theta} \xi_t^{\nu} - h_{t+1}] + \gamma \ln h_{t+1}\} \tag{5}$$

Maximization with respect to n_t yields the first order condition:

$$\frac{1}{n_t} = \frac{\phi(h_t)\lambda\xi_{t+1}}{(\xi_{t+1} - h_{t+1})(1-n_t)} \tag{6}$$

while first condition with respect h_{t+1} amounts to:

$$\frac{1 - \theta\xi_{t+1}/h_{t+1}}{\xi_{t+1} - h_{t+1}} = \frac{\gamma}{h_{t+1}} \tag{7}$$

The expression above can be solved for h_{t+1} getting:

$$h_{t+1} = \frac{\gamma + \theta}{\gamma + 1}\xi_{t+1} \equiv \alpha\xi_{t+1} \tag{8}$$

Hence by α has been denoted the health share of consumption. Now it is straightforward to compute optimal consumption as:

$$c_{t+1} = \frac{1-\theta}{1+\gamma}\xi_{t+1} \equiv (1-\alpha)\xi_{t+1} \tag{9}$$

As expected, health investment is increasing (while c_{t+1} is decreasing) in γ and θ: this is not surprising since γ represents the preference for health over consumption; θ is related

[8]Here is assumed that the only specific cost of learning is in terms of time, or equivalently that monetary cost of children education is included in parents consumption.

to the productivity effect of health[9]. Because the utility function is homothetic, optimal health-consumption ratio (χ) is independent on ξ_{t+1}, and amounts to:

$$\frac{h_{t+1}}{c_{t+1}} = \frac{\gamma + \theta}{1 - \theta} \equiv \chi \tag{10}$$

Now, substituting (8) for h_{t+1} into (6), the optimal amount of time to devote to learning can be computed:

$$1 - n_t = \frac{\phi(h_t)\lambda(\gamma + 1)}{\phi(h_t)\lambda(\gamma + 1) + (1 - \theta)} \equiv \Delta \tag{11}$$

Denoting by Δ_i the partial derivative of Δ with respect to variable i, we have the following proposition:

Proposition 1 *Under private regime, time devoted to learning activities is increasing in life expectancy, returns from instruction, return from health and preference for health over general consumption:*

$$\Delta_\phi > 0, \quad \Delta_\lambda > 0, \quad \Delta_\theta > 0, \quad \Delta_\gamma > 0$$

Proof: The signs follow by taking derivatives of (11) ■

The sign of each derivative is as expected. The fraction of time that agents decide to invest in learning is increasing in the portion of second period they expect to live, $\phi(h_t)$: if agents have an higher life expectancy, returns from human capital accumulation are worth more and hence it is preferable to consume less leisure. If λ increase, learning time is more productive so that it is optimal to increase it. As far as θ and γ are concerned, a direct and an indirect effect can be detected, operating both in the same direction. The indirect effect works through (8) by increasing h_t and so $\phi(h_t)$: when individuals live longer because they grew in healthier familiar environment they discount less the future and are more wiling to invest in human capital accumulation. The direct effect is instead related to the positive impact of γ and θ on health share of outcome: from (6) it can be seen that the optimal leisure/learning ratio is increasing in consumption share of income, which amounts to $1 - \alpha \equiv \frac{1-\theta}{\gamma+1}$. If health share decreases, individuals prefer to have more leisure; the opposite happens when an increase in γ or θ causes health share to increase as well. The intuitive reason is as follows: either because returns from being healthy are higher (θ) either because health is more appreciated (γ) individuals devote more income to health investment. In order to keep the optimal technical rate of

[9] In this model elasticity of health with respect to income is constant and equal one: health is a normal good, but neither a primary neither a luxury good. This can be a good approximation since "health" is considered in a broad sense. The increased taste for health that possibly happened in Western Economies in recent decades can be interpreted in this context as an increase in γ.

substitution between inputs in human capital technology, a higher amount of education is demanded as well.[10]

4 Public Funding

In the public funding regime, health investment is provided by government, which levies a proportional tax τ_t on wage income determined through majority voting, so that the common health investment is given by:

$$h_t = \bar{h}_t = \tau_t \bar{\xi}_t \tag{12}$$

where $\bar{\xi}_t$ denotes the average human capital.

Hence in public regime constraint (4) is modified in:

$$\xi_{t+1}(1 - \tau_{t+1}) = c_{t+1} \tag{13}$$

Since there is a common level of health, under public system human capital is accumulated following:

$$\xi_{t+1} = \psi(1 - n_t)^{\lambda} \bar{h}_{t+1}^{\theta} \xi_t^{\nu} \tag{14}$$

The agents maximize

$$U = \ln n_t + \phi(\bar{h}_t)[\ln c_{t+1} + \gamma \ln \bar{h}_{t+1}]$$

with respect to n_t, c_{t+1} and τ_{t+1} subject to (13) and (14). As in GR, we assume that individuals are aware of (12), but they take as given $\bar{\xi}_t$, since they see as negligible their contribution to average human capital. Substituting the constraints in the expression above and maximizing with respect to n_t the following first order condition is obtained:

$$\frac{1}{n_t} = \frac{\phi(\bar{h}_t)\lambda}{1 - n_t} \tag{15}$$

that can be solved to find optimal time devoted to human capital:

$$1 - n_t = \frac{\phi(\bar{h}_t)\lambda}{1 + \phi(\bar{h}_t)\lambda} \equiv \tilde{\Delta} \tag{16}$$

As in the private case, we derive a proposition for comparative statics on $\tilde{\Delta}$:

[10] in GR the optimal amount of time is $\lambda/(1/2 + \lambda)$. It is easy to see this result as a particular case of (11) when $\gamma = 1$ and $\phi(h_t) = 1$, as in fact is in their model, and $\theta = 0$: in their framework human capital investment is a mere bequest and it affects productivity only starting from the next generation. Hence the technology parameter (i.e. θ) describing marginal returns of this investment is not taken into account.

Proposition 2 *Under public regime, time devoted to learning activities is increasing in life expectancy and returns from instruction; there is no effect of returns from health and preference for health but the one through lifespan:*

$$\tilde{\Delta}_{\phi} > 0, \quad \tilde{\Delta}_{\lambda} > 0, \quad \tilde{\Delta}_{\theta} = 0, \quad \tilde{\Delta}_{\gamma} = 0$$

Proof: The signs follows by taking derivatives of (16) ■

The optimal tax rate is the result of the optimization with respect to τ_{t+1} of the following function:

$$\ln[(1-\tau_{t+1})\xi_{t+1}] + \gamma \ln(\tau_{t+1}\bar{\xi}_{t+1}) \tag{17}$$

First order condition gives:

$$\frac{-\xi_{t+1} + (1-\tau_{t+1})\theta\xi_{t+1}/\tau_{t+1}}{(1-\tau_{t+1})\xi_{t+1}} + \frac{\gamma}{\tau_{t+1}} = 0 \tag{18}$$

From the equation above is possible to find the optimal tax rate, which turns out to be:

$$\tau^* = \frac{\theta+\gamma}{1+\theta+\gamma} \tag{19}$$

As in GR, because of the functional forms chosen for utility, τ^* is independent of individual income[11] and constant over time. By (12) and (19), we have health investment under public regime:

$$\bar{h}_{t+1} = \frac{\theta+\gamma}{1+\theta+\gamma}\bar{\xi}_{t+1} \tag{20}$$

In GR, fraction of income devoted to health is 1/2 in both regimes. This happens because in their model human capital investment affects productivity only starting from next generation; parents have altruism investing in human capital for their children, but this bequest is totally independent on technology of human capital accumulation. Here, if an agent invest in health, she directly improves her productivity and she is aware of human capital technology. GR's result can be seen as a particular case when $\gamma = 1$ (as they implicitly assume) and $\theta = 0$.

4.1 Dynamics

Under private regime, equation (3) describes dynamics for human capital accumulation. Substituting for $(1-n_t)$ and h_{t+1} their optimal values and arranging, a dynamic equation for human capital evolution can be found:

$$\xi_{t+1} = [\psi\alpha^{\theta}\Delta^{\lambda}\xi_t^{\nu}]^{\frac{1}{1-\theta}} \tag{21}$$

Inside Δ appears $\phi(h_t)$ which depends on health investment at time t, optimally chosen equal to $\alpha\xi_t$. Since agents' longevity is a function of health at time t which is a fraction

[11]Hence this tax rate is preferred not only through majority voting, but also unanimously.

of human capital at time t, a further channel of persistence is introduced in human capital law of motion, that is not at work when only education is considered because it comes from a peculiar effect of health. In order to have an explicit dynamic equation a functional form for $\phi(h_t)$ should be chosen. For the sake of simplicity $\phi(\cdot)$ is chosen according to (4.2a). In Appendix A can be found results that would be obtained by using the specification as in (2). Then Δ, that represents optimal time devoted to learning, amounts to:

$$\Delta(\xi_t) = \frac{\beta\lambda\xi_t}{\left(\beta\lambda + \frac{1-\theta}{\gamma+1}\right)\xi_t + \frac{1-\theta}{\gamma+\theta}} \tag{22}$$

It is possible to highlight the role of health share and health-consumption ratio, rewriting in a more compact form the expression above:

$$\Delta(\xi_t) = \frac{\beta\lambda\xi_t}{(\beta\lambda + 1 - \alpha)\xi_t + \chi^{-1}}$$

Actually, however $\phi(\cdot)$ is chosen between (2) and (4.2a), Δ exhibits the following properties: it is between zero and 1, non decreasing in ξ_t, concave in ξ_t, with $\lim_{\xi\to 0} = 0$ and $\lim_{\xi\to\infty} < 1$. Moreover it is increasing in health expenditure measured as share of income (α) and as ratio over consumption (χ). These results are supported by empirical evidence: an increasing health expenditure often came with an increase in time devoted to human capital accumulation.

Under public funding dynamics of human capital can be studied by substituting optimal leisure from (16) and tax rate from (19) in (14):

$$\xi_{t+1} = \psi\tau^{*\theta}\tilde{\Delta}^{\lambda}\bar{\xi}_{t+1}^{\theta}\xi_t^{\nu} \tag{23}$$

Inside $\tilde{\Delta}$ appears $\phi(\bar{h}_t)$ which depends on health investment at time t, optimally given by $\tau^*\bar{\xi}_t$. As in private case, in order to give an explicit dynamic equation a functional form for $\phi(\cdot)$, expression (4.2a) is used:

$$\tilde{\Delta}(\bar{\xi}_t) = \frac{\beta\lambda\tau^*\bar{\xi}_t}{(\beta\lambda + 1)\tau^*\bar{\xi}_t + 1} \tag{24}$$

Anyhow $\phi(\cdot)$ is chosen between (2) and (4.2a), Δ is smaller than 1, non decreasing in ξ_t, concave and with $\lim_{\xi\to 0} = 0$ and $\lim_{\xi\to\infty} < 1$.

5 Homogeneous Agents

In this section we compare results from the previous section under the assumpiton that initial distribution of human capital is degenerate and agents are endowed with the same amount of human capital, so that each individual is equal to the average one.

With undifferentiated agents the following proposition on health share of income holds:

Proposition 3 *If agents are homogeneous the share of income devoted to health is greater under private regime.*

Proof: Follows by comparison of (8) and (20) when $\xi = \bar{\xi}$ ∎

This comes from the fact that the effect on own human income (and hence income) is seen as negligible in the public regime. Notice that the two system yields the same share when $\theta = 0$.

Time devoted to human capital under private and public regime is ruled respectively by (11) and (16). If agents are homogeneous then $\xi_t = \bar{\xi}_t$, $\forall t$. The following result is readily available:

Proposition 4 *If agents are homogeneous, time devoted to learning activities is higher under private regime:*

$$\Delta > \tilde{\Delta}$$

Proof: See Appendix B ∎

This means that if agents are undifferentiated then in private regime individuals devote more time to learning thus boosting growth: this happens essentially because under public funding is imposed to everybody the same level of health investment, and so h_{t+1} is seen only in private regime as a fraction of agent's own human capital ξ_{t+1}, while in public system it is a function of average human capital $\bar{\xi}_{t+1}$. This is by construction an advantage of private system in terms of growth rate. If a policy of mandatory schooling is enforced, then it would be possible to set time allocated to human capital investment as in (11) also in public system, and this source of difference in human capital law of motion would be eliminated, even if private regime would remain preferable from a welfare viewpoint[12].

When agents are homogeneous, then $\xi_t = \bar{\xi}_t$ in every period. In other words it is possible to consider just a representative agent, focusing only on growth issues, abstracting from distributive ones. Before comparing the dynamics of the two systems of funding it is interesting to notice the implications that the properties of Δ and $\tilde{\Delta}$ add to the dynamics of human capital: Δ and $\tilde{\Delta}$ are between 0 and 1, non decreasing in ξ_t, concave in ξ_t and with $\lim_{\xi_t \to 0} = 0$ and $\lim_{\xi_t \to \infty} < 1$. Because they share same properties, the analysis of the dynamics of human capital can be restricted without loss of generality to one

[12] An alternative issue to explore could be a public system where government collects money levying a proportional income tax and then give back to the agents a common transfer, with the agents free of choosing their optimal health investment. In this case the budget constraint becomes: $\xi_{t+1}(1-\tau_{t+1}) + z_{t+1} = c_{t+1} + h_{t+1}$; where $z_{t+1} = \tau_{t+1}\bar{\xi}_{t+1}$. This extension is considered in Appendix D.2

of them, e.g. Δ. Since Δ depends positively on the cumulative factor ξ_t, it increases the marginal return of cumulative factor and augments the persistence of dynamics. This effect is not at work in GR's model since in their context the discount factor is independent from human capital.

This further contribution of ξ_t through Δ on ξ_{t+1} is anyway only temporary since, as human capital is cumulated, Δ tends to a finite value, less than one, so that in the long run returns of cumulative factor are determined only by $\nu/(1-\theta)$. The intuition behind that is that at low levels of longevity, an increase in health investment, not only makes workers more productive, but makes their children more likely to live longer, thus encouraging them to devote more time to learning because they discount less the future. Then productivity and hence income increase as well, so that agents can afford to augment also their health investment. As income increases, also health investment increases, but the improvements in life expectancy augment less than proportionally, so that the effect on discounting is progressively less important and at the limit it vanishes.

In other terms, it is possible to think at effect of health on optimal learning time as a positive externality that a generation has into the next because of its choice related to health care. Of course this can be seen also in a reverse perspective: if the environment conditions worsen (that can be broadly seen as a negative investment in health) then the externality is negative and next generation has an incentive to discount more the future.

This effect is particularly interesting also because it may give rise to poverty and underdevelopment traps. In the long run dynamics are governed by $\nu/(1-\theta)$. If the ratio is less than 1, i.e. $\nu + \theta < 1$, but "closed enough" to 1, then, under compatible but reasonable values for parameters, there are two positive equilibria: the greatest one is stable, the smallest one unstable[13]. Fig.2 provides a graphical example for such a case.

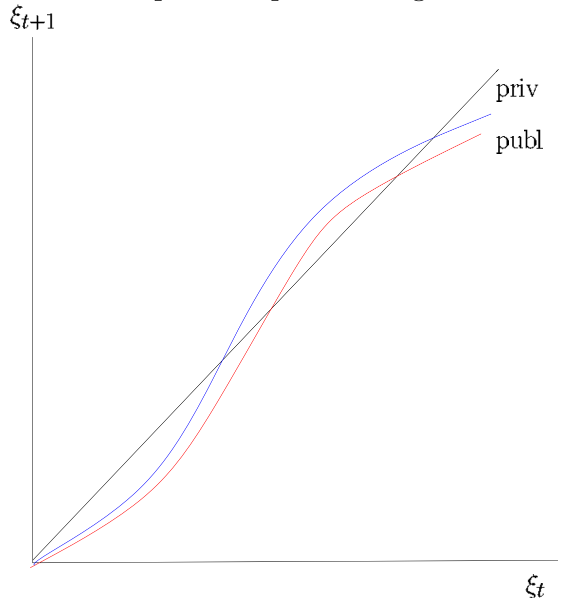
At low levels of human capital, the discount effect described above plays a role, increasing temporary returns of human capital. This can induce a temporary convexity on the map of ξ if the sum $(\nu+\theta)$ is not "too low" and hence the map may intersect the 45 degrees line from below, in other words an unstable steady state is possible.

Moreover, if the economy starts with ξ_0 less than this unstable equilibrium, it will converges to zero, i.e. zero is a catching point for human capital dynamics[14]; while if ξ_0 is bigger than the unstable equilibrium, then the economy will converge to the other positive equilibrium, that is stable and greater. The intuition behind this result comes from the evidence of virtuous and vicious circles that has been mentioned in Section 1: even if a constant fraction of income is devoted to health, when human capital is too

[13] If parameters are such that no intersection happens at positive value of human capital, the only (stable) equilibrium is the trivial one: 0; while if parameters are such that even for small value of ξ, the map is above the 45 degrees line, then there is only one positive (stable) steady state and the zero equilibrium is unstable. In Section 7 is found that values of parameters implying poverty trap are a relevant empirical case.

[14] 0 is a *catching point* for x if, for small x, $g(x) < x$, where $g(x)$ denotes the map function of x.

Figure 2: A Human Capital Map with long run decreasing returns



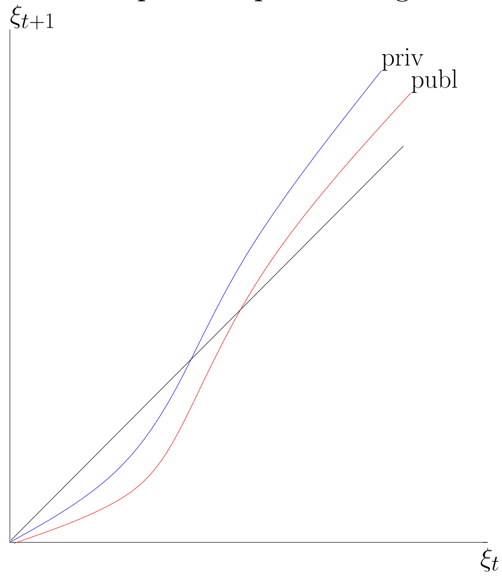
low, life expectancy is low too and agents has no incentive to devote a lot of time to learning because they discount heavily the future, but in this way human capital cannot grow enough to escape the trap and economy is condemned to a situation of poverty and insufficient health, that in its model counterpart is represented by the zero equilibrium. On the contrary, if initial conditions are high enough, virtuous circles made by more health - less discounting - more income, can take place.

Constant long run returns in cumulative factor, i.e.: $\nu + \theta = 1$, are a sufficient (but not necessary) condition to have convexity in the first part of the map of ξ. Then in the long run the map turns into a straight line. Under compatible and reasonable values of parameters the map intersects the 45 degrees line (see Fig. 3). In this case there is a stable equilibrium in zero (catching point) and an unstable equilibrium at the positive intersection point. If the economy starts with ξ_0 grater than the unstable equilibrium, it experiences long run balanced growth; otherwise if it is lower the poverty traps mechanism described above takes place. It is worthwhile noticing that even under long run constant return of scale in cumulative factor, endogenous growth is not granted and historical conditions may determine the pattern of the economy.

Finally, with increasing returns, i.e.: $\nu + \theta > 1$, under compatible but reasonable values for parameters, two equilibria exist: the zero stable equilibrium (catching point) and the positive unstable equilibrium, above which growth of human capital is explosive[15]. It

[15]The other possible scenario is that zero equilibrium is not a catching point: in this case it is the only steady state, it is unstable and for any positive ξ_0 the economy experiences long run explosive growth.

Figure 3: A Human Capital Map with long run constant returns



is worthwhile noting that neither the increasing returns condition is able alone to grant long run growth.

Now we go back to comparison between the two regimes of health funding. Under the assumption of homogeneous agents, in order to assess which one performs the higher growth, it is only needed to compare (21) and its public counterpart (23), considering that $\xi = \bar{\xi}$, $\forall t$. Since $\alpha > \tau^*$ for $\theta \in (0;1)$, a sufficient condition for private system to bring about higher growth is $\Delta > \tilde{\Delta}$. Indeed it is proofed in Appendix A that with homogeneous agents the inequality holds for both specifications of $\phi(\cdot)$ that have been considered. Differently from GR's setting, even if a policy of mandatory schooling is chosen the gap in favour of private system remains because learning time is not the only source of difference between the two regimes. The other channels is that in private funding a bigger portion of income is devoted to health, which is an input of human capital technology.

Denoting by $\underline{\xi}_{pr}$ and $\underline{\xi}_{pu}$ the smallest positive (unstable) steady state in private and public regimes respectively, and by ξ^*_{pr}, ξ^*_{pu} the greatest positive (stable) steady state, results can be summarized as in the following proposition:

Proposition 5 *With homogeneous agents, in the long run we have:*

- *If $\nu + \theta < 1$ and two positive steady states exist: $\underline{\xi}_{pu} > \underline{\xi}_{pr}$; $\xi^*_{pr} > \xi^*_{pu}$. Private system yields a greater stable steady state.*
- *If $\nu + \theta = 1$ and a positive steady state exists: $\underline{\xi}_{pu} > \underline{\xi}_{pr}$. The long run constant growth rate under private funding is greater.*

- *If $\nu + \theta > 1$ and a positive steady state exists:* $\underline{\xi}_{pu} > \underline{\xi}_{pr}$.

Proof: Results follow from inspection of (21), (23), and Fig.2, Fig.3 ■

Some remarks can be done: first, whatever the assumption on returns, if $\underline{\xi}_{pr}$ and $\underline{\xi}_{pu}$ exist, then the latter is always greater, meaning that under public system a higher ξ_0 is required to escape poverty trap. Second, theoretically it is possible that positive steady states and long run growth exist only for private system, since human capital map in the private case is always above the map in the public case. Third, these results hold under the hypothesis of homogeneity; keeping this in mind they are rather intuitive: in public system less implications of health investment than in private system are internalized, this introduces in the former regime an inefficiency, so that the latter performs better both in terms of growth and conditions to avoid poverty traps.

6 Heterogeneous Agents

If agents are heterogeneous, the proposition that everyone devotes more time to learning under private regime no longer necessarily holds. Instead we have:

Proposition 6 *If agents are heterogeneous, then time devoted to learning activities is greater under public regime for those agents for whom:*

$$\phi(\bar{h}_t) > \frac{\gamma + 1}{1 - \theta}\phi(h_t) \equiv \frac{1}{1 - \alpha}\phi(h_t)$$

Proof: See Appendix B ■

Independently of how $\phi(\cdot)$ is chosen, public regime is more likely to imply an higher time investment in learning if health share of income is low. This can be seen as direct consequence of the fact that under public regimes less effects of health are internalized. Those effects have the property to amplify the difference in lifespan due to the two regimes: if health becomes more important either because of preferences (γ) either because of change in productivity (θ), private system becomes more likely to favor time investment in human capital more than what public system does.

In order to examine in further details this issue it is necessary to choose an explicit form for $\phi(\cdot)$. We considered an explicit form as in (4.2a). Results from specification as in (2) can be found in Appendix A. Using this expression a condition on relative human capital can be derived. The amount of time devoted to learning is higher in public regime for agents whose human capital is not too high compared to the average:

$$\frac{\xi_t}{\bar{\xi}_t} < \frac{(1-\theta)(1+\gamma)}{(1+\gamma)(1+\theta+\gamma)+\bar{\xi}_t(\theta+\gamma)^2} \equiv \frac{\tau^*}{\chi(1+\bar{\xi}_t\tau^*\alpha)}$$

This relations means that if an agent's human capital is relatively low enough, then her children would choose to devote more time to human capital accumulation under the public regime. This happens because if children's life expectancy depends on health investment that their parents can only privately afford it would be low and they would have less incentives to invest. Instead under public regime their lifespan may benefit from the public health investment which is greater than what they could afford otherwise. At low level of human capital this effect may overcome the advantage of private regime in terms of internalizations that has been highlighted above.

Keeping constant the ratio $\xi_t/\bar{\xi}_t$, the condition above becomes more and more difficult to be verified as the level of average human capital increases. This happens essentially because of the concave shape of $\phi(\cdot)$: improvements in life expectancy are smaller as the health investment is augmented.

It may be worth noticing that however $\phi(\cdot)$ is chosen, the condition to have that public regime induces an agent to devote more time to human capital accumulation is more likely to be verified if the agent's human capital is lower compared to the average, if optimal tax rate is high, and if health share of income is low. The reason behind that is that is necessary to create a differential in favor of health investment in public regime: this is affected positively by τ^* and $\bar{\xi}_t$ as can be seen from (12), and negatively by private health share of income since it includes the advantage in terms of internalization that characterizes private funding. Notice also that both τ^* and α depend positively on γ and θ, so an increase in these parameters have contrasting effects on conditions that assess whether public regime promotes learning more than private system.

We now come to the dynamics in the heterogeneous case. Things are more complicated than in GR since here current human capital ξ_t plays a further role through Δ making less simplifications possible from the assumption of log-normality, and comparing the two regimes in terms of evolution of average log-human capital is not straightforward.[16] Furthermore there is also to assess what happens when a poverty traps arises.

As far as public system is concerned, applying logarithms to (23) and calling $\ell \equiv \ln \xi_t$, one obtains:

$$\ell_{t+1} = \nu\ell_t + \lambda \ln \tilde{\Delta}(\bar{\xi}_t) + \theta \ln \bar{\xi}_{t+1} + \ln(\psi\tau^{*\theta}) \tag{25}$$

We assume the following:

[16] Moreover ξ_{t+1} in public system depends on its concurrent average $\bar{\xi}_{t+1}$, while in GR's model appears the lagged average.

Assumption 1 *The initial distribution of human capital is log-normal.*

If human capital is log-normally distributed at time t with mean μ_t and variance σ_t^2, then human capital at time $t+1$ is also log-normally distributed with mean μ_{t+1} and variance σ_{t+1}^2, where

$$\mu_{t+1} = \nu\mu_t + \lambda \ln \tilde{\Delta}(\bar{\xi}_t) + \theta \ln \bar{\xi}_{t+1} + \ln(\psi\tau^{*\theta}) \quad (26)$$

$$\sigma_{t+1}^2 = \nu^2\sigma_t^2 \quad (27)$$

As in GR's model, agents with low income experience higher growth rates than agents with high income because ξ_{t+1}/ξ_t is a decreasing function of ξ_t in the public regime since $0 < \nu < 1$, hence inequalities tend to disappear. Expression (26) is complicated by the fact that the logarithm of the average human capital ($\ln \bar{\xi}$) and the average of the logarithm (μ) appear both at time $t+1$. The assumption of log-normality has an important implication in this sense since it provides a relation between μ_{t+1} and $\bar{\xi}_{t+1}$[17]:

$$\bar{\xi}_t = \exp\left(\mu_t + \frac{\sigma_t^2}{2}\right) \quad (28)$$

Using this property and (27), (26) can be rewritten as:

$$\mu_{t+1} = \frac{1}{1-\theta}\left[\nu\mu_t + \frac{\theta\nu^2}{2}\sigma_t^2 + \lambda \ln \tilde{\Delta} + \ln(\psi\tau^{*\theta})\right] \quad (29)$$

where $\tilde{\Delta}$ is a function of μ_t and σ_t^2 once $\bar{\xi}_t$ has been replaced.

A result of our reference model can now be replied:

Proposition 7 *If two economies under public regime start with the same per-capita income, the economy with lower inequality experiences higher growth*

Proof: See Appendix C ■

As far as the private system is concerned, it can be noticed that by applying logarithm to (21) one gets:

$$\ell_{t+1} = \frac{\nu}{1-\theta}\ell_t + \frac{1}{1-\theta}[\lambda \ln \Delta(e^{\ell_t}) + \ln(\psi\alpha^\theta)] \quad (30)$$

Applying the expected value operator, the following expression is obtained:

$$\mu_{t+1} = \frac{\nu}{1-\theta}\mu_t + \frac{\lambda}{1-\theta}E[\ln \Delta(e^{\ell_t})] + \frac{1}{1-\theta}\ln(\psi\alpha^\theta) \quad (31)$$

Since Δ is a function of ℓ_t, the expression above cannot be reduced to a simple linear relation between μ_{t+1} and μ_t. Even if it is not straightforward to compute the law of

[17] Moreover, for the log-normal distribution, the Gini coefficient depends only on standard deviation σ_t. Hence under the assumption of log-normality, inequalities may be measured also through Gini coefficient without adding other parametric specifications to the distribution.

motion for the variance in private regime in a short form as (27) for the public regime, it should be easy to see that in every period it will be greater than its public counterpart: the coefficient of ℓ_t in (30) is greater than in public system (see (25)), moreover Δ is increasing in ℓ_t and positively correlated with the first term in ℓ_t, thus for sure inequalities in private system cannot decline faster than under public regime.

In order to have an idea about dynamics of growth and inequalities with heterogeneous agents, some properties of Δ and $\tilde{\Delta}$ may be of help: they are increasing in ξ_t and $\bar{\xi}_t$ respectively but at decreasing rate so that the limits are finite and less than one. The next paragraphs provide further details examining cases with or without poverty traps from a theoretical point of view, while Section 7 through calibration of parameters provides some simulative examples for the case of constant long run returns.

6.1 Inequalities and growth without poverty traps.

In this paragraph it is assumed that poverty traps do not arise. In terms of map graph this means that either the human capital map function does not cross in its convex tract the 45 degrees line either that even the lowest endowed agent is above the poverty trap threshold (the unstable steady state). Note that these assumptions do not preclude any case about long run returns $(\nu + \theta)$. We first consider decreasing returns in the long run. The following proposition on inequality dynamics and income growth holds

Proposition 8 *Under long run decreasing returns, inequalities decreases in both regimes but faster in the public one. Though the long run positive steady state is higher under private regime, during the transition the public system may feature higher growth.*

Proof: The results on inequalities dynamics follows from analysis in Section 6 comparing (25) and (30). As far as the transition result is concerned, notice that by comparing (29) and (31) can be seen that private regime has an advantage due to $\psi\tau^{*\theta} < \psi\alpha^{\theta}$,[18] on the other side as long as agents are heterogeneous, μ_{t+1} in public system has a positive term in σ_t^2. Moreover $\ln(\tilde{\Delta}(\bar{\xi}_t))$ can be greater than $E[\ln(\Delta(\xi_t))]$: since functions ln and Δ are both concave, the inequality holds if $\Delta(\cdot) = \tilde{\Delta}(\cdot)$, but by continuity it still holds for some $\Delta(\cdot) > \tilde{\Delta}(\cdot)$. Anyway since in the long run the human capital distribution is degenerate under both regimes, the ultimate result coincide with the homogeneous case, with a greater steady state under private system. ■

Intuitively: if society is very unequal, public health funding may induce some agents to accumulate more human capital than they would do with private funding because in public system their expected lifespan is higher since it benefits of an amount of health expenditure that they could not afford by themselves. The reduction in discount

[18]Notice that if $\theta = 0$, the two expressions are equal, so the lower is θ the lower is this advantage of private regime.

factor they may benefit is bigger than the potential reduction the richest agents loose because improvements in life expectancy comes at a decreasing rate. Since returns are decreasing in the long run, who starts with a lower level of human capital grows faster, thus inequalities decrease progressively (even quicker under public regime). As society becomes more equal the condition under which public system provides more incentive to human capital accumulation becomes more and more restrictive and it is growth-enhancing to switch to private regime, where more health implications are internalized. Inequalities continue to decrease but at a slower rate, until they finally vanish in the long run.

Proposition 9 *Under long run constant returns, i.e.* $\nu + \theta = 1$*, inequality declines only under public system whereas in the private regime they initially increase and then stay steady. The long run growth factor is higher under private regime.*

Proof: In public system there is again progressive reduction in inequalities, so that the analysis of the homogeneous case for the endogenous balanced growth can be applied. Under private system inequalities are kept constant in the long run, but in the short run they increase since there is a temporary convexity in the map function, hence long run inequalities are higher than the initial ones.
Long run growth factor in private regime amounts to: $(\psi\alpha^{\theta}\Delta^{*\lambda})^{1/(1-\theta)}$ where Δ^* denotes the limit of Δ for ξ_t that tends to infinite[19] . It can be easily seen that this growth factor is greater than the one of public regime: $(\psi\tau^{*\theta}\tilde{\Delta}^{*\lambda})^{1/(1-\theta)}$. ■

Hence at least in the long run there is a trade-off between growth and equality in income distribution. Which system should be preferred from a welfare viewpoint cannot be said a-priori without knowing the weight that the policy maker assigns on equality. Under decreasing returns instead in the long run private system is pareto-superior since inequalities vanish in both systems but private regime yields higher steady state income. During the transition also with constant returns the growth factor can be higher in public regime, as shown by some examples in Section 7.

Finally, for the sake of completeness, with increasing returns, both systems grow at increasing rates, anyway in private system rate of growth increases (explodes) faster. As far as inequalities are concerned, the two systems bring about two opposite results: under public system they vanish, while in private regime they increase more and more since returns are higher for whom who begin with higher initial conditions.

[19]With $\phi(\cdot)$ specified as in (2a) this limit is $\frac{\beta\lambda}{\beta\lambda+\frac{1-\theta}{\gamma+1}}$ for Δ and $\frac{\beta\lambda}{\beta\lambda+1}$ for $\tilde{\Delta}$, while under (2) it amounts to $\frac{\lambda\bar{\phi}}{\lambda\bar{\phi}+\frac{1-\theta}{\gamma+1}}$ for Δ and $\frac{\lambda\bar{\phi}}{\lambda\bar{\phi}+1}$ for $\tilde{\Delta}$.

6.2 Growth and inequalities with poverty traps

Under decreasing returns, in private regime is likely to take place a bi-polarization in human capital distribution: those who start below the threshold level (which is also the smallest steady state) converges to the zero equilibrium, while those who start above converges to the stable steady state equilibrium. Inequalities are between groups, but not within. Denoting by x the fraction of agents that are at the beginning in the poverty trap basin, the final average income is[20]: $(1-x)\xi^*_{pr}$. The steady state variance of the bipolar distribution can be easily computed: $\sigma^2 = (1-x)x\xi^*_{pr}$. Hence at aggregate level inequalities do not vanish although decreasing returns because poverty trap prevents the convergence mechanism to fully take place. Inequality increases in ξ^*_{pr} because it augments the gap with the zero equilibrium; also it increases with x for $0 < x < 1/2$, then decreases: if exactly one half of the population is involved in poverty trap there is a perfect bi-polarization and hence variance is at its maximum level.

A similar analysis can be done under constant returns, with the difference that people who avoid the poverty trap, then grow everyone at rate $(\psi\alpha^\theta\Delta^{*\lambda})^{1/(1-\theta)}$. The economy growth rate is a weighted average between this rate and zero: $(1-x)(\psi\alpha^\theta\Delta^{*\lambda})^{1/(1-\theta)}$. This means that long run growth rate depends on initial condition: if two economies where health is privately financed have the same deep parameters but they start with a different portion of agents involved in poverty traps (for example because of a different distribution, mean or variance), then one will experience more growth than the other. If per-capita income is high the economy that is more equal has more chance to grow more; the opposite, if per-capita income is low.

This happens because in private regime what determines long run growth is the portion of agents that are outside the basin of attraction of poverty trap: if average income is above that threshold and if the distribution is more concentrated around the mean then a greater portion of agent may escape poverty trap; on the other side, if average income is very low, then growth may take place only if someone is outside the poverty trap basin, and this is more likely to happen if the variance of the distribution is high. Note that in this model without intra-generational spillover this does not help at all who has been caught by the zero catching point.

That has important implications in terms of inequalities, since the distribution becomes bi-polar with only one group growing. Hence there is an ever-increasing disparity between the two groups and hence also for the economy; while the richest group experiences long run growth, there is a part of population that lives in stagnant misery. Differently from GR this happens even if long run returns are not increasing, and is a direct consequence of the endogenous poverty trap.

The analysis can be extended also to the increasing returns case, where anyway it is not possible to define a long run growth rate since it is ever-increasing. It should be

[20]For the sake of simplicity it is assumed that no agent is at the beginning in the unstable steady state.

remarked that increasing returns implies that inequalities augment not only between the "trapped" and the growing group but also within the latter.

How do these conclusions change under public system?

It is expectable that some results of the analysis with no poverty trap can still hold: public regime reduces inequalities more than private system, whatever the assumption on long run returns; if an economy is characterized by high inequality then public system may yield to higher growth, at least for some periods. With poverty traps the fact that health is provided at common level could help several agents to escape form the basin of attraction of the zero equilibrium if human capital distribution exhibits some skewness (e.g. if average human capital is above the median human capital), as it happens for the log-normal distribution, and average human capital is high enough. This happens because ξ_{t+1} depends on a mix of average and agent-specific human capital. On the other side the risk is that, especially at low level of per-capita income, neither the relatively more endowed agents, who would own enough resources to escape from the poverty trap, can succeed to grow because they are forced to rely on a too low common health provision[21].

Even if closed-form solutions are hard to find, it is possible to provide examples showing how income distribution moves for different initial conditions. In order to do that anyway, parameters have to be calibrated; what is done in the next section.

7 Simulation

7.1 Calibration

First of all is necessary to approximately fit the periods to real years. We can assume to split the life at the age of 33. In the very first part of life agents' decisions are taken by parents, then, e.g. at 13, they have to choose how much to study in order to accumulate human capital, in this sense leisure includes all what is not learning. After they work for 30 years from 33 to 63 years old. Lifespan can then be expressed as $33 + \phi(h_t)30$: the bottom level for life expectancy match well enough what the data show (Fig.3). The top level is a little bit underestimated, thus suggesting the possibility of extending the model by including a third age where agents spend money saved during the adulthood. For the sake of simplicity only two period are assumed and $\phi(\cdot)$ is chosen according to (4.2a). All that implies that setting $\beta = 1$ can be reasonable.

The health share of income α can be used to calibrate γ and θ. Empirically health expenditure share of GDP is between 4.3% that represents the average for Africa and

[21] If this is the case, beginning from private system in order to foster growth and then switching to public system to decrease inequalities when average income is high enough to make this policy effective can be considered.

11.7% for Europe, with a maximum in USA of 13.9%, (data from World Health Report [23]). Actually, because of the simpiflyng assumption of no physical capital, the figure that α should match should be the share of health expenditure over total consumption, or at least over GDP net of physical investment. In this sense calibrating $\gamma = 0.05$ and $\theta = 0.1$ implies a reasonable value for health expenditure share of 14.3%, while τ^* amounts to 13%. This choice of θ is taken accounting also for the effects of θ on productivity, based on Bloom *et al.* [4] and Sachs *et al.* [22][22].

As far as the learning coefficient λ is concerned, the main implication is that in general the higher the value of λ the more human capital map is likely to exhibit an initial convexity: in order to give count of poverty traps λ should not be too low. De la Croix and Doepke [7] set a conservative estimate in 0.6, that is kept here. Moreover it is assumed to have balanced growth in the long run; this implies that $\nu + \theta = 1$, so that $\nu = 0.9$.[23] The productivity parameter ψ can then be fixed such to have a long run reasonable growth rate, i.e. 2% by year, given the choice of θ, γ, λ and the the number of years for which individuals produce: computation gives $\psi = 3.531$. A relevant issue is that in presence of poverty traps the average growth rate depends on historical conditions because a portion x of population are caught by the poverty trap. Hence the growth rate on which is based the calibration of ψ should be intended as the rate of growth for the group of agent starting after the unstable steady state.

7.2 Simulated dynamics

Fig.4 shows the map functions in this benchmark framework under the assumption of homogeneous agents. Results predicted in section 5 are confirmed: private system grows more than public system and the basin of attraction to the poverty trap equilibrium is larger under the public regime. This implies that in a society of perfectly equal agents there is a region for initial conditions such that public system leads to poverty whereas private regime can sustain endogenous growth.

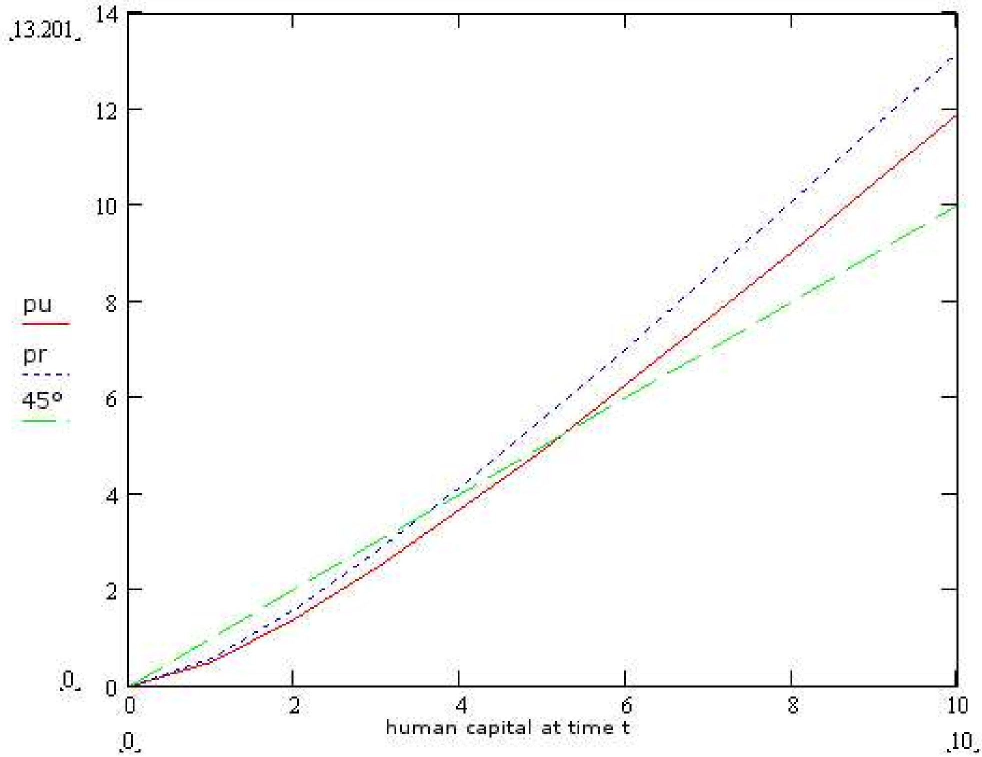
What happens now if heterogeneity is allowed? Some numerical experiments have been run keeping the benchmark parametrization and varying the mean and the variance of the normal distribution of log-human capital, for a population of 150 individuals. In what follows we report the main regularities that emerge.

First of all, we assess the comparison for a situation in which average initial log-human capital is high enough so that *basically nobody is involved in the poverty trap mechanism*,

[22]In fact, it may be a little bit too high but I think it can be reasonable for this benchmark representation. If a lower θ is chosen, the main qualitative effect is to make flatter the human capital map.

[23]Maybe it is worthwhile reminding that in this model ξ_t is not a simple externality of human capital on its next level but more broadly characterizes persistence in the level of knowledge. Therefore the chosen value for ν does not seem exaggerated.

Figure 4: Human capital map with **homogeneous agents**.



for example $\mu = 8$.[24] When variance is zero, i.e. homogeneous case, private system brings about higher growth. The variance is then fixed to 4. It can be observed in Fig. 5 that public system reduces inequalities whereas in private regime they rise at the beginning and then remain constant. From (28), $\ln(\bar{\xi}_t)$ in the two economies can be computed, and then a comparison of how growth factors evolve over time can be represented in Fig. 6.

Simulation Regularities 1 *When no poverty trap occurs, the following regularities emerge:*

- *private regime yields initially higher growth rates but, provided the initial distribution is sufficiently unequal, during the transition public regime may experience higher growth factors*
- *if two societies start with the same per-capita income but different variance then the more equal society grows more, whatever the regime.*

[24]This value is near to the logarithm of current US $ GNI per capita at world level, which according to Atlas method in 2002 amounted to 8.54 (World Bank data).

Figure 5: Dynamics of variance for $\mu = 8$ and $\sigma^2 = 4$.

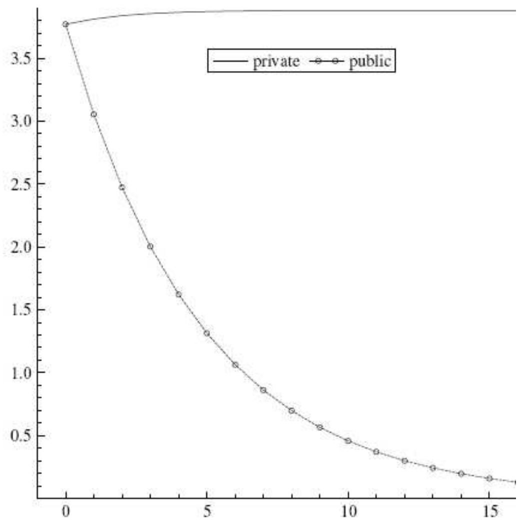
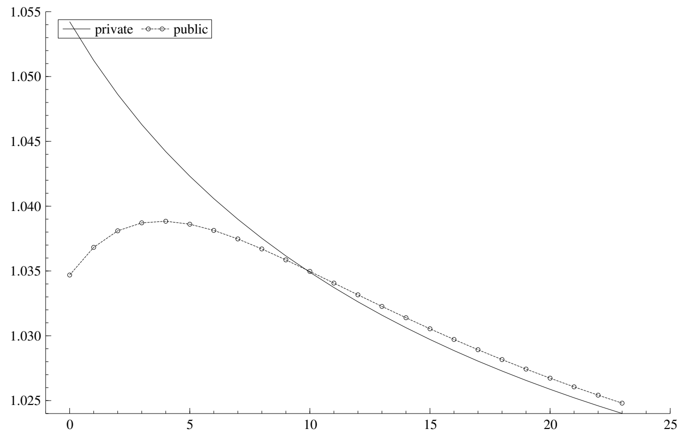


Figure 6: Dynamics of growth factors of log-per capita income for $\mu = 8$ and $\sigma^2 = 4$.



- *if two societies start with the same per-capita income but different variance, then the period during which public system exhibits higher growth factors is longer in the more unequal society.*

These regularities are now commented, with the help of graphics from the simulations. If two economies start with the same μ, private system leads at the beginning to higher growth rates in per capita-income: the growth enhancing mechanism related to health effects on life expectancy works immediately, then as this effect decreases because of properties of Δ private regime's growth factor converges exponentially to its long run value. Note that the rise in the initial variance contributes to augment per-capita income in a lognormal distribution, (see (28)[25]). In public system the mechanism is slower because a redistributive channel is at work, but when the growth slows down it takes more time to decrease as well. It is interesting to observe that inequality introduces a hump-shape in the growth rate of log per capita income in public economy: if initial inequality is big enough, redistribution slows down growth immediately, but then while it continues to reduce inequalities the growth-enhancing mechanism operating through $\tilde{\Delta}$ is triggered and the public economy converges to its long run path where agents are progressively more and more homogeneous. Hence during transition public regime may experience higher growth factors than private regime. By varying the initial inequalities can be seen that to have this result initial variance should not be too low. For small values of σ^2 growth factor in private economy is always greater.

All that suggests that in equal society, the private regime should be preferable, whereas in unequal society the public system, though statically less efficient from a growth viewpoint in a dynamical sense can combine lower inequalities and higher growth rates.

The second point of simulation regularities 1 puts in evidence that, ceteris paribus, a lower inequality can be desirable also from a growth perspective. It has been claimed in Section 6 that if two public system start with the same μ but different σ^2 then the more equal society grows more; simulations show that the same happens if two private economies are concerned, even if this difference turns out to be less than in public regime. The intuitive reason is that in the more equal society, more agents are closed to the mean income and thus their average will be higher than the unequal society's because of concavity of Δ and $\tilde{\Delta}$, and of human capital accumulation technology with respect to each input.[26]

Finally, if two societies start with the same per-capita income but a different degree of inequality, then the length of the period over which public system yields greater growth factors than private system is larger in the more equal society. This just comes by

[25] In fact μ_t tends to grow more, at least at the beginning, under the public regime, but this effect is overcome by the simultaneous decrease in variance in public regime compared to the variance of the private system.

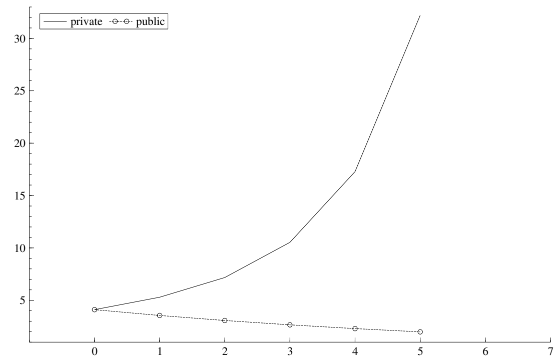
[26] Since Δ is "less concave" than $\tilde{\Delta}$ (in private system agents sees higher returns from devoting time to learning), then such result is mitigated in the private regime: this explains the last part of second point in Simulation Regularities 1.

extending findings from the first point. Not only the time interval turns out to be longer the greater is initial inequality, but also the quantitative gap turns out to be larger at every step.

If the initial distribution is chosen with a lower mean or a higher variance, than *some agents may be involved in the poverty trap mechanism.* This has a direct impact on inequalities dynamics since in private regime inequalities become explosive, as explained in section 6.2. As shown in Fig. 7 in public system there is a continuous link between the poor and the rich since they share the same health provision which depends on average income, thus contrasting the endogenous force toward increasing inequalities which characterizes private regime.

In order to assess growth issues under poverty traps, the initial distribution is centered on a relatively low level, e.g. $\mu = 4$, so that some agents are involved in poverty trap because they are in the catching point basin. In private regime every agent that has an endowment inferior to the poverty trap threshold converges to the zero equilibrium, while the others grow. What happens from an economic growth viewpoint? Fig.8 is relative to growth factors in this case.

Figure 7: Dynamics of variance for $\mu = 4$ and $\sigma^2 = 4$.

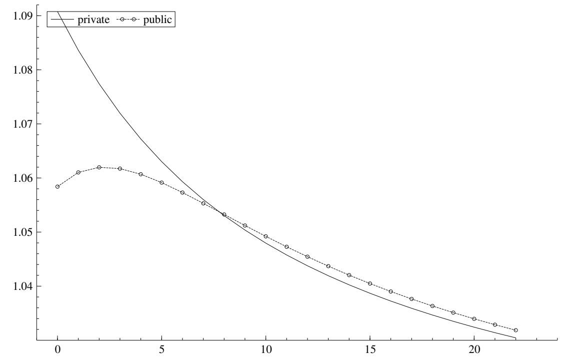


The following regularities emerge:

Simulation Regularities 2 *When poverty trap occurs:*

- *at given variance the decline in μ increases the initial advantage of private system, but usually the interval for which public system yields higher growth factors increases as well*
- *at given mean, the initial gap is lower the lower is initial inequality. An increase on initial inequality does not affect substantially the length of the interval over which public system grows faster, but affect the size of this gap.*

Figure 8: Dynamics of growth factors of log-per capita income for $\mu = 4$ and $\sigma^2 = 4$.

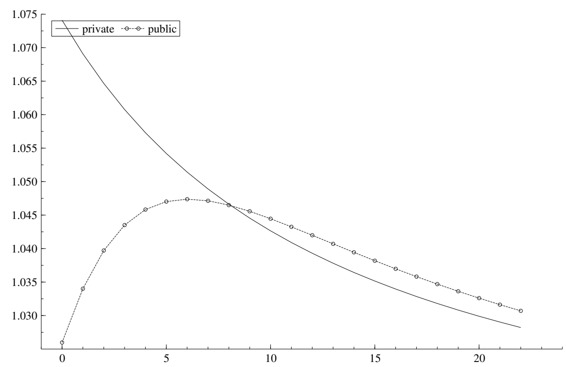


- *if initial per capita income is very low, then public regime can grow faster than private regime in the future only if initial inequality is high enough.*

About the first point we add that only in the case agents are perfectly equal private system always experiences higher growth rates. When there are inequalities, in this situation with low income the stickiness in motion of public regime's human capital increases, and through this channel the period over which this regime yields higher growth is longer.

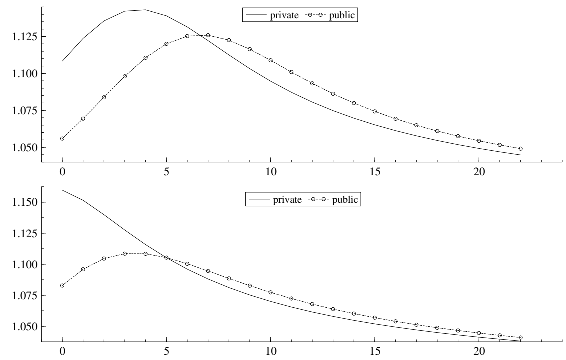
About the second point, dynamics shown in Fig.9 when $\sigma^2 = 6$ roughly exhibit the behavior illustrated before for Fig. 6. Varying σ^2 has no significant effect on the length of the interval for which public system overcomes private system in terms of growth factors, but affects the impact size of this overtake: if a society is more unequal at the beginning, then when public system's growth rate overcomes private system's growth rate, it does that quantitatively more.

Figure 9: Dynamics of growth factors of log-per capita income for $\mu = 4$ and $\sigma^2 = 6$.



About the third point, we consider what happens by choosing for μ a value such that in the homogeneous case only private regime escapes poverty trap. By looking at Fig.4 and considering that μ is expressed in logarithmic scale, the chosen value is 1.65. As expected if agents are perfectly equal, only private system is able to escape poverty trap[27]. Then heterogeneity is introduced by increasing progressively the initial variance, running several times the experiment for each value of σ^2. By continuity it turns out that in order to have public system able to escape poverty trap σ^2 should not be too low. At this very low level of income, where poverty trap is the rule more than the exception for many agents, initial inequality has an important impact on timing of those dynamics: in private system the growth mechanism is triggered soon for those who can afford it, while in public system it may take more time, because it comes through a redistributive mechanism, but then it involves more people. The higher is σ^2 the longer is the interval for which growth rate is higher in public regime than in private regime, nevertheless if σ^2 is high enough, human capital begins to move more quickly and the relative gap between the two regimes shrinks. A comparison of growth dynamics for two different σ can be found in Fig. 10.

Figure 10: Dynamics of growth rates of log-per capita income with $\mu = 1.654$ and $\sigma^2 = 0.5$ (top) and $\sigma^2 = 2$ (bottom).



Concluding, in poor societies, it seems reinforced the trade-off between the immediate advantage in growth terms yielded by the private regime and the advantage in a longer perspective yielded from the public regime with inequality reduction and higher growth rates. If society is very poor, but income distribution is relatively not unequal the model predicts that private regime is preferable; if instead income distribution is sufficiently unequal, the public regime can bring about longer periods of sustained growth. Beginning with a private regime can be preferred when only under private regime some agents may escape the poverty trap and hence accumulate human capital.

[27]It is worthwhile noticing that for this very low average initial value also growth rate of private regime exhibits an hump-shape.

8 Conclusions

It has been presented a model of economic growth in an OLG setting to focus on differences in terms of growth and inequalities between private and public health funding. Health enters directly in utility function, but it is important also indirectly because it improves productivity and increases life expectancy thus reducing the discount factor and encouraging human capital investment. In private system agents decide in a decentralized way how much to spend in health, while in public regime health is provided by government and financed through proportional taxation with the tax rate determined through majority voting.

The fact that health is provided in the same amount implies that less positive effects of health are internalized since agents see their contribution to average income as negligible. The common provision is by construction a weak point of public regime also from a welfare viewpoint. Nevertheless is worth emphasizing that even providing private system with such advantages in terms of internalization, it does not follow that private regime yields anyhow sustained higher growth rates than public regime.

Public system turns out to be more effective in reducing inequalities and differently from the reference model of Glomm and Ravikumar may induce poor agents to devote more time to learning since their survival probability may be higher because of public provision of health care. Since improvements in lifespan comes at decreasing rate, this positive effect is bigger than the negative one borne by the richest.

It is found that if initial income inequality is not too large, private system yields always higher growth, but if initial variance is large enough private system may be overcome for several periods by public system in terms of growth factors. Simulations show that it is more likely to happen as initial inequality increases.

The model is able to account for endogenous poverty traps. They are due to the temporary convexity induced in human capital map by the effect of lifespan on human capital investment. When a poverty trap is at work, private regime exhibits ever increasing inequalities in the endogenous growth case. Generally the growth factor turns out to be initially higher in private regime, but then it can be overcome by public system's for several periods.

The health effect on lifespan acts like an inter-generational externality. It could be interesting to consider also intra-generational externalities, since improving general health conditions reduce the risk of contracting contagious epidemics and diseases. In this framework it could be analysed the problem of health provision from the point of view of a social planner, with the pubic regime that could internalize a factor that is exogenous at private level, thus reducing introducing a trade off between internalization of effects in the public regime (due to intra-generational externality) and in the private regime (due to impact on own productivity).

Finally, another line along which the model can be extended is to include in the analysis a third period, the old age, which realistically should represent another important component of health demand and give rise to an increase in health expenditure, without having necessarily positive effects on growth, as happened in Western Economies. This would also provide a natural framework in this OLG setting to include savings and physical capital accumulation. All these possible extensions represent suggestions for further research.

References

[1] L. Aguiar-Conraria. Public versus private schooling in an endogenous growth model. NIPE Working Paper 7, 2004.

[2] R. Asa and F. Pueyo. Endogenous longevity, health and economic growth: a slow growth for a longer life? *Economics Bulletin*, 9:1–10, 2001. No. 3.

[3] R. Barro. Health and economic growth. Paper prepared for the Pan American Health Organization available at http://www.paho.org/English/HDP/HDD/barro.pdf, 1996.

[4] D. E. Bloom, D. Canning, and J. Sevilla. The effect of health on economic growth: A production function approach. *World Development*, 32(1):1–13, January 2004.

[5] S. Chakraborty. Endogenous lifetime and economic growth. *Journal of Economic Theory*, 2004.

[6] S. Chakraborty and M. Das. Mortality, human capital and persistent inequality. *Journal of Economic Growth*, 10(2):159–92, 2005.

[7] D. de la Croix and M. Doepke. Public versus private education when differential fertility matters. *Journal of Development Economics*, 93:1091–1113, 2004.

[8] D. de la Croix and O. Licandro. Life expectancy and endogenous growth. *Economic Letters*, 65:255–263, 1999.

[9] A. Deaton. Health, inequality, and economic development. *Journal of Economic Literature*, (41):113–158, 2003.

[10] N. Devlin and P. Hansen. Health care spending and economic output: Granger causality. *Applied Economic Letters*, 8 (8):561–564, 2001.

[11] R. Fogel. *New findings on secular trends in nutrition and mortality: Some implications for population theory*, volume 1A. Elsevier Science, The Netherlands, 1997. in M.Rosenzweig and O. Stark "Handbook of population and family economics".

[12] V. Fuchs. *The Health Economy.* Harvard University Press, Cambridge, MA, 1986.

[13] O. Galor and D. Mayer. Food for thought: Basic needs and persistent educational inequality. Paper prepared for the Pan American Health Organization held in Washington, October 4, 2002.

[14] G. Glomm and B. Ravikumar. Public versus private investment in human capital: Endogenous growth and income inequality. *Journal of Political Economy*, 100 (No.4):818–834, 1992.

[15] K. Hosoya. Tax financed government health expenditure and growth with capital deepening externality. *Economics Bulletin*, 5, No. 14:1–10, 2003.

[16] P. Howitt. Health, human capital and economic growth: a schumpeterian perspective. Paper prepared for the Pan American Health Organization, held in Washington, October 4, 2002.

[17] M. Leung and Y. Wang. Endogenous health care and life expectancy in a neoclassical growth model. University of Hong Kong, January 2003.

[18] D. Mayer Foulkes. Human development traps in mexico. Division de Economia, 2004.

[19] Organizations of United Nations (UN). Human development report. Technical report, 1990. United Nations Development Programme.

[20] Micronutrient Initiative UNICEF. Vitamin and mineral deficiency: A global progress report. Technical report, UNICEF, 2004. available at www.micronutrient.org.

[21] A. Van Zon and J. Muysken. Health and endogenous growth. *Journal of Health Economics*, 20:169–185, 2001.

[22] World Health Organization (WHO). Macroeconomics and health: Investing in health for economic development. Technical report, 2001. Report of the Commission on Macroeconomics and Health.

[23] World Health Organization (WHO). The world health report. 2004.

A Alternative specification of $\phi(\cdot)$

If $\phi(\cdot)$ is chosen according to (2), we have for Δ:

$$\Delta(\xi_t) = \begin{cases} \frac{\lambda a\left(\frac{\gamma+1}{\gamma+\theta}\right)^{1-\epsilon}}{\lambda a\left(\frac{\gamma+1}{\gamma+\theta}\right)^{1-\epsilon}+\frac{1-\theta}{\gamma+\theta}\xi_t^{-\epsilon}} & \text{if } \xi_t < \hat{\xi}_t \\ \frac{\lambda\bar{\phi}}{\lambda\bar{\phi}+\frac{1-\theta}{\gamma+1}} & \text{otherwise} \end{cases} \tag{32}$$

where the threshold level $\hat{\xi}_t$ is defined as $(\bar{\phi}/a)^{1/\epsilon}(\gamma+1)/(\gamma+\theta)$.

The expression above can be rewritten more shortly as:

$$\Delta(\xi_t) = \begin{cases} \frac{\lambda a\alpha^{\epsilon-1}}{\lambda a\alpha^{\epsilon-1}+\chi^{-1}\xi_t^{-\epsilon}} & \text{if } \xi_t < \hat{\xi}_t \\ \frac{\lambda\bar{\phi}}{\lambda\bar{\phi}+\chi^{-1}} & \text{otherwise} \end{cases}$$

In the public regime we have:

$$\tilde{\Delta}(\bar{\xi}_t) = \begin{cases} \frac{\lambda a\bar{\xi}_t^{\epsilon}}{\lambda a\bar{\xi}_t^{\epsilon}+(\tau^*)^{-\epsilon}} & \text{if } \bar{\xi}_t < \tilde{\hat{\xi}}_t \\ \frac{\lambda\bar{\phi}}{\lambda\bar{\phi}+1} & \text{otherwise} \end{cases} \tag{33}$$

where the threshold level $\tilde{\hat{\xi}}_t$ is defined as $(\bar{\phi}/a)^{1/\epsilon}(\gamma+\theta+1)/(\gamma+\theta)$.

In order to assess conditions under which an agent devotes more time to learning under public regime, three possible situations may arise:

- Both average and agent specific human capital are below the threshold levels of health investment[28]. In this case a relation between ξ_t and $\bar{\xi}_t$ can be found, stating a condition for public funding to enhance learning investment more than private system:

$$\frac{\xi_t}{\bar{\xi}_t} < \frac{(1-\theta)^{1/\epsilon}}{(1+\gamma+\theta)(1+\gamma)^{\frac{1-\epsilon}{\epsilon}}} \equiv (1-\alpha)^{1/\epsilon}\frac{\tau^*}{\alpha}$$

- Agent's health investment in private system is below the threshold level while public health investment is above. In this case public funding yields higher time for learning if

$$\xi_t < \frac{\gamma+1}{\gamma+\theta}\left(\frac{\bar{\phi}(1-\theta)}{a(\gamma+1)}\right)^{1/\epsilon} \equiv \frac{1}{\alpha}\left(\frac{\bar{\phi}(1-\alpha)}{a}\right)^{1/\epsilon}$$

[28]These threshold levels expressed in terms of human capital are $\frac{\gamma+1}{\gamma+\theta}\left(\frac{\bar{\phi}}{a}\right)^{1/\epsilon}$ for ξ_t under private regime and $\frac{\gamma+\theta+1}{\gamma+\theta}\left(\frac{\bar{\phi}}{a}\right)^{1/\epsilon}$ for $\bar{\xi}_t$ under public regime.

- If agent health investment in private regime is above the threshold level then learning is higher in private system whatever the average human capital. Intuitively this happens because the lifespan is already at its maximum so there is no chance that public system may make up the gap with private regime by increasing life expectancy.

B Proof of Propositions 4 and 6

If agents are homogeneous then $\Delta > \tilde{\Delta}$, whatever the specification of $\phi(\cdot)$.

$$\Delta > \tilde{\Delta} \quad \Leftrightarrow \quad \frac{\phi(h_t)\lambda(\gamma+1)}{\phi(h_t)\lambda(\gamma+1)+(1-\theta)} > \frac{\phi(\bar{h}_t)\lambda}{1+\phi(\bar{h}_t)\lambda}$$

The inequalities above hold if and only if:

$$\phi(h_t)\lambda(\gamma+1)+\phi(h_t)\phi(\bar{h}_t)\lambda^2(\gamma+1)-\phi(h_t)\phi(\bar{h}_t)\lambda^2(\gamma+1)-\phi(\bar{h}_t)\lambda(1-\theta)>0$$

$\Leftrightarrow$

$$\phi(h_t)\lambda(\gamma+1)-\phi(\bar{h}_t)\lambda(1-\theta)>0$$

The inequality above always holds if agents are homogeneous, i.e. $\xi=\bar{\xi}$, since $\gamma,\theta\in(0;1)$ and

$$h_t=\frac{\gamma+\theta}{\gamma+1}\bar{\xi}_t>\frac{\theta+\gamma}{1+\theta+\gamma}\bar{\xi}_t=\bar{h}_t$$

C Proof of Proposition 7

Proof of comparison between two public economies in Section 6: *If two public economies start with the same per-capita income then if $\sigma_t'^2>\sigma_t^2$ then $\bar{\xi}_{t+1}>\bar{\xi}'_{t+1}$.*

Since $\bar{\xi}_{t+1}=\mu_t+\sigma_t^2$, using (29) and (27), $\bar{\xi}_{t+1}>\bar{\xi}'_{t+1}$ if and only if

$$\frac{\nu}{1-\theta}\mu_t+\frac{\nu^2}{2(1-\theta)}\sigma_t^2>\frac{\nu}{1-\theta}\mu'_t+\frac{\nu^2}{2(1-\theta)}\sigma_t'^2$$

Now, because $\bar{\xi}_t=\bar{\xi}'_t$ we have:

$$\frac{\nu}{1-\theta}(\mu_t-\mu'_t)=\frac{\nu}{2(1-\theta)}(\sigma_t'^2-\sigma_t^2)>\frac{\nu^2}{2(1-\theta)}(\sigma_t'^2-\sigma_t^2) \quad \text{since } \nu<1.$$

Same reasoning holds for every t that follows.

D Further issues

D.1 Childhood Health Productivity

Human capital is likely to be positively affected by the health condition in childhood. If the human capital accumulation function (3)is modified in

$$\xi_{t+1} = \psi(1-n_t)^{\lambda} h_t^{\theta} \xi_t^{\nu} \tag{34}$$

for private system and

$$\xi_{t+1} = \psi(1-n_t)^{\lambda} \bar{h}_t^{\theta} \xi_t^{\nu} \tag{35}$$

for public regime. This specification is even closer to Glomm and Ravikumar's model. The main implication is that in the maximization problem the only control variable that affects ξ_{t+1} is the learning time $(1-n_t)$ since h_t is chosen by parents. In private regime, optimization of (1) subject to (4) and (34) yields

$$h_{t+1} = \frac{\gamma}{1+\gamma}\xi_{t+1} \equiv \alpha' \xi_{t+1} \tag{36}$$

In public regime, optimization of (1) subject to (13) and (35) yields:

$$\tau'_{t+1} = \frac{\gamma}{1+\gamma} \tag{37}$$

It can be noticed that $\tau' = \alpha'$ and that $\alpha = \alpha' = \tau' = \tau^*$ if $\theta = 0$: considering childhood health on one side increases the impact of the inter-generational externality, on the other side implies the loss of an advantage of private regime in terms of internalization: whatever the regime the choice of health has no direct impact on income, since only the human capital of next generation is affected. Hence the share of income devoted to health in both systems is less than in the case with adulthood health in human capital accumulation. This reduction anyway is bigger under private regime and the share of income devoted to health turns out be the same.

The condition under which an agent devotes more time to learning under public regime than under private becomes:

$$\frac{\xi_t}{\bar{\xi}_t} < \frac{\alpha'}{\gamma(1+\alpha'^2\bar{\xi}_t)}$$

The threshold is bigger than in the adulthood health case, hence under this specification is likely to be greater the number of agents investing more time in human capital under public regime.

If agents are homogeneous private regime yields higher stable steady state or long run growth factor (depending on long run returns). It can be seen by comparing the laws of motion of human capital accumulation under private and public regimes, which are respectively:

$$\xi_{t+1} = \psi\alpha'^{\theta}\Delta'^{\lambda}\xi_t^{\nu+\theta} \tag{38}$$

$$\xi_{t+1} = \psi\alpha'^{\theta}\tilde{\Delta}'^{\lambda}\bar{\xi}_t^{\theta}\xi_t^{\nu} \tag{39}$$

If agents are homogeneous (i.e. $\xi_t = \bar{\xi}$) private regime yields to higher stable steady state or long run growth factor (depending on long run returns). Anyway now the gap is completely determined by Δ and $\tilde{\Delta}$, and the advantage of public regime, even if lower than before, is confirmed. As in Glomm and Ravikumar, a policy of mandatory school is able to cancel out this source of advantage for private regime.

Analyzing the human capital dynamics, one can see that now the model is closer to Glomm and Ravikumar's prediction. For private regime:

$$\mu_{t+1} = (\nu + \theta)\mu_t + \lambda E[\ln \Delta(e^{\ell_t})] + \ln(\psi\alpha'^{\theta}) \tag{40}$$

For public regime:

$$\mu_{t+1} = (\nu + \theta)\mu_t + \frac{\theta}{2}\sigma_t^2 + \lambda \ln \tilde{\Delta}(e^{\mu_t + \sigma_t^2}) + \ln(\psi\alpha'^{\theta}) \tag{41}$$

The law of motion of log per capita income under private regime becomes:

$$\ln \bar{\xi}_{t+1} = (\nu + \theta)\mu_t + \frac{\sigma_{t+1}^2}{2} + \lambda E[\ln \Delta(e^{\ell_t})] + \ln(\psi\alpha'^{\theta}) \tag{42}$$

In public regime:

$$\ln \bar{\xi}_{t+1} = (\nu + \theta)\mu_t + \frac{\theta + \nu^2}{2}\sigma_t^2 + \lambda \ln \tilde{\Delta}(e^{\mu_t + \sigma_t^2}) + \ln(\psi\alpha'^{\theta}) \tag{43}$$

With respect to Glomm and Ravikumar's model, here the concavity of log function and $\Delta(\cdot)$ plays a role in favour of public system: $\ln(\tilde{\Delta}(\bar{\xi}_t))$ can be greater than $E[\ln(\Delta(\xi_t))]$: since functions ln and Δ are both concave, the inequality holds if $\Delta(\cdot) = \tilde{\Delta}(\cdot)$, but by continuity it still holds for some $\Delta(\cdot) > \tilde{\Delta}(\cdot)$, on the other side we know that in private regime σ_{t+1}^2 is greater than $(\theta + \nu)^2$, as it is in our reference model, hence it cannot be said unambiguously if public system is more likely to bring about higher per-capita income than in our reference.

Compared to the benchmark specification (adulthood health), the main qualitative result is that the gap between private and public regime is reduced. Considering both childhood and adulthood health in human capital production function can be done in this way:

$$\xi_{t+1} = \psi(1 - n_t)^{\lambda} h_{t+1}^{\delta\theta} h_t^{(1-\delta)\theta} \xi_t^{\nu} \tag{44}$$

where $\delta \in (0; 1)$. Maximization yields analogous results of the case with only adulthood helth, since health share of income now amounts to $\frac{\gamma+\delta\theta}{\gamma+1}$ and $\tau^* = \frac{\gamma+\delta\theta}{\gamma+1+\delta\theta}$. Then the dynamics are between the two extreme cases, the closer to the benchmark case the higher is δ.

D.2 Different public funding scheme

In the benchmark model is assumed that health is provided in a common level by government. What happens if instead government uses a common transfer proportional to average income with the households free of choosing their optimal amount of health? In other terms, the budget constraint for a household becomes: $\xi_{t+1}(1-\tau_{t+1})+z_{t+1} = c_{t+1}+h_{t+1}$; where $z_{t+1} = \tau_{t+1}\bar{\xi}_{t+1}$. If it is rewritten as: $\xi_{t+1} + \tau_{t+1}(\bar{\xi}_{t+1} - \xi_{t+1}) = c_{t+1} + h_{t+1}$.

It is easy to see that all the households with income below the average would prefer the maximum possible tax rate, i.e. $\tau = 1$, on the other side all the household with income above the average would prefer a the lowest possible tax rate, i.e. $\tau = 0$. Except for voters with average income (who are indifferent regarding the tax rate) preferences are single peaked. If we assume that no voter has exactly the average income, the median voter theorem can be applied. Hence the optimal tax rate is the one preferred by the median voter. If the original distribution is such that the median voter has less than the average income (as in the log-normal distribution) we have that the tax rate chosen under majority voting (but no longer unanimously) is $\tau = 1$. Then for every household the budget constraint becomes: $\bar{\xi}_{t+1} = c_{t+1} + h_{t+1}$. Maximization of (1) under that constraint and (3) implies that househould devotes to health a share of their income equal to $\frac{\gamma}{1+\gamma}$, that is smaller not only of α but also of τ^*, the health share of income when health is provided publicly at a common level.

Hence using this kind of transfers is detrimental for growth. This happens because if the initial distribution is unequal enough, the majority voting triggers a very powerful redistributive mechanism so that every household has basically the same disposable income; nevertheless this redistribution implies the loss of internalization of all effects of health investment in current income since the individual contribution to average income is seen as negligible by the agents. Even the public system with common provision of health implies a higher health expenditure, because agents consume out of their income and they are aware of the contribution that their health conditions has on it.